Equilibrium of Soil Grains at the Threshold of Movement by Wind

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ABSTRACT

This paper, based on wind tunnel studies, presents an analysis of the nature and magnitude of forces on soil grains at the threshold of their movement by wind. Forces of drag, lift, and gravity were analyzed in relation to each other.

The equilibrium between these forces and the soil grains was found to be influenced by the diameter, shape, and immersed density of the grains, the angle of repose ϕ' of the grains with respect to the mean drag level of the fluid, the closeness of packing η of top grains on the sediment bed, and the impulses of fluid turbulence T_D and T_L associated with drag and lift, respectively. All those factors were measured and related with actual and theoretical forces involved.

New approaches to measurement of η, ϕ', T_D and T_L are presented. Analyses indicate that the magnitude of pressure impulses of both T_D and T_L is statistically distributed according to the somewhat skewed normal error law. The ratio of mean pressure to standard deviation σ was constant for any size of grains or fluid velocity and could be expressed by equation $\sigma = c\bar{P}$ in which $\bar{P}$ is the mean pressure of lift or drag and c is a constant which was found to have a mean value of 0.49.

[1]Contribution from Soil and Water Conservation Research Division, ARS, USDA, Kansas Agr. Exp. Sta. cooperating. Department of Agronomy contribution No. 621. Presented before Div. I, Soil Science Society of America, Aug. 7, 1958, at Lafayette, Ind. Received April 27, 1959. Approved July 31, 1959.

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Factors that govern movement of soil materials by fluids have been investigated extensively because of the key role they play in soil erosion. These factors are intricate and not entirely understood.

Although a considerable number of investigations have been made on the mechanics of movement of soil material by water, much less has been done on the phenomenon of movement of soil material by wind. While the basic laws of fluid motion and pressure are essentially the same for air as for water, some notable differences exist in their manifestations.

The study reported in this paper was made to determine the nature and magnitude of forces induced by wind movement at the threshold of movement of soil grains.

SYMBOLS

d = Diameter of spherical grain on a bed.

F_c = Critical drag on the top grain on a bed.

F_c' = Critical drag per unit horizontal area occupied by the top grain.

$\overline{F}$ = Mean drag on unit horizontal area of the top grain on a bed.

τ_c = Critical drag on unit horizontal area of the whole bed.

$\bar{\tau}_o$ = Mean drag on unit horizontal area of the whole bed.

$\bar{\tau}_c$ = Critical mean drag on unit horizontal area of the whole bed.

L_c = Lift on the top grain at the critical drag F_c on the grain.

$\overline{L}$ = Mean lift on unit horizontal area of the top grain on a bed.

L_c' = Critical lift on unit horizontal area of the whole bed.

$\overline{L}'$ = Mean lift on unit horizontal area of the whole bed.

$\overline{L}_c'$ = Critical mean lift on unit horizontal area of the whole bed.

$\overline{P}_L$, $\overline{P}_D$ = Mean pressure of lift and drag, respectively.

ρ = Density of fluid.

ρ' = Difference in density of grain and fluid. (Immersed density of grain.)

σ = Standard deviation.

g = Acceleration of gravity.

f_o, f_o' = Frequency of occurrence of zero pressure of drag and lift, respectively.

ϕ = Angle of repose of the grain on a bed with respect to its center of gravity.

ϕ' = Angle of repose of the grain on a bed with respect to the mean drag level of the fluid.

η = Ratio of mean drag and lift per unit area on the whole bed to mean drag and lift per unit area on the top grain moved by the fluid, ($\bar{\tau}_o/\overline{F}$ and $\overline{L}'/\overline{L}$).

T = Ratio of "maximum" to mean drag and lift on the sediment grain, assuming the ratio to be the same for both drag and lift.

T_D = Ratio of "maximum" to mean drag on the grain. It is taken as $(P_D + 3\sigma/\overline{P}_D)$.

T_L = Ratio of "maximum" to mean lift on the grain. It too is taken as $(\overline{P}_L + 3\sigma)/\overline{P}_L$.

v = Mean velocity of the fluid.

V_* = Mean drag velocity of the fluid.

V_{*t} = Mean threshold drag velocity of the fluid—the minimum drag velocity required to produce a continuous movement of top grains.

REVIEW OF LITERATURE

The forces required to initiate movement of the top grains on a bed of wind-drift material are known as the *critical, threshold*, or *incipient* forces. All three terms are found in the literature and have the same meaning.

Jeffries (7) carried out an analysis of transport of sediments by streams and concluded that fixed particles in a stream of water or air are subjected to lifting forces due to specific pressure distributions on their surfaces. He also concluded that the momentum effect along the direction of flow (form drag) on the particles is negligible. Einstein and El-Samni (5) measured a significant lift force in a stream of water which tends to pull sediment away from the surface and concluded that lift is the major moving force of bed-load. An oscillogram published from their study seems to show that the maximum lift pulsations are from 2.25 to 2.5 times the mean.

Fage, as reported by White (9), found with an ultramicroscope that the maximum velocity of pulsations of turbulent flow in the direction of flow in a rough, square pipe is 1.5 times the average velocity. Since pressure varies as the square of velocity within the range of the experiment, it implies that the maximum stress of pulsations is 2.25 times the average. White (9) observed maximum impulses of fluid velocity above the sediment grains about twice the average and concluded that the maximum drag pressure is about four times the average. Contrary to Jeffries (7) and Einstein and El-Samni (5), he claimed that lift on a bed of sediment grains is negligible. Kalinske (8) too assumed that lift is negligible and that the maximum drag pressure is four times the average, presumably relying on White's inference from velocity fluctuation near the grains. However, this inference is valid only if the velocity of flow against the grain surface is known. The velocity against the sediment grains is largely unknown. It is indicated in this paper that the projected average fluid velocity is zero at some distance above the average surface of the fixed sediment grains. Therefore, the conclusion that pressure on the sediment grain varies as the square of fluid velocity above the grain may be invalid. In fact, Einstein and El-Samni (5) suggested that "—pressures are the basic variables in the statistical description of turbulence in the vicinity of the wall, rather than the velocity." Therefore, it was decided in the present study to measure directly the magnitude of impulses of horizontal and vertical pressures on spheres similar to erodible grains lying on the surface.

From direct and indirect measurements, Chepil (3) found that average lift on hemispherical roughness elements ranging from 0.16 to 5.08 cm. in height was about 0.85 that of drag. The ratio of lift to drag on the elements remained virtually constant for any drag velocity of the fluid, size of elements, and depth of the fluid boundary beyond a relatively shallow depth.

Both White (9) and Kalinske (8) presumed that all of the drag $\bar{\tau}_o$ is taken up by the topmost grains moved by the fluid. On this assumption, the so-called "packing coefficient" η was determined by counting the grains lying on top of the bed, computing the largest horizontal cross-sectional area of the grains, and dividing this area by the total horizontal area of the bed. However, exact determination of the coefficient in this manner is impossible since all sorts of gradations between complete exposure and virtually complete embedding of the surface grains occur. Any distinction between grains that take up the drag and grains that do not is purely arbitrary. Some grains take up more drag than others and some none at all, depending on their elevation in the surface layer. A new approach for determining η is presented in this paper.

In their publication on forces acting on individual particles on a granular bed, Ippen and Verma (6) affirmed that "Nothing is known in detail as yet concerning the turbulent pressure fluctuations near the bed." This paper in part presents some progress made on this phase of the problem.

DESCRIPTION AND THEORETICAL ANALYSIS OF THE PROBLEM

Movement of a soil grain is initiated when the pressures caused by the moving fluid against the grain overcome the force of gravity. Three types of pressure on a grain may result from movement of the fluid. One is a positive pressure against the part of the grain facing into the direction of fluid motion resulting from the impact of the fluid against the grain. This is called *impact*, or *velocity pressure*. For velocities causing the initiation of movement of a soil grain, the velocity pressure varies directly as the square of fluid velocity and its magnitude is the force per unit of grain area normal to the direction of movement of the fluid.

The second type is a negative pressure on the lee side of the grain known as *viscosity pressure.* Its magnitude depends on the fluid's coefficient of viscosity, density, and velocity.

The third type of pressure is a negative pressure on top as compared to the bottom of the grain caused by the Bernoulli effect. Wherever the fluid velocity is speeded up, as at the top of the soil grain, the pressure (measured transverse to the general direction of fluid motion) is reduced. This is called the *static, isotropic,* or *internal pressure.*

The impact or velocity pressure on a soil grain lying on the ground is known as *form drag,* and pressure due to viscous shear in the fluid close to the surface of the soil grain as *skin friction drag.* Their combined force is known as *total drag.* No separation of the two forces is made in this study. The total drag henceforth in this paper will be referred to as drag. The drag on the top grain at the threshold of its movement is due to the pressure difference against its windward and leeward sides. The arrow in figure 1 indicates the general direction in which it acts. Although it acts on the grain normal to the direction of fluid flow, it is generally expressed in terms of mean force per unit horizontal area of the whole bed.

A decrease in static pressure at the top of the grain as compared to that at the bottom causes a *lift* on the grain. It is determined by, but is not the same as the pressure difference against the top and the bottom halves of the grain. The arrow indicates the general direction in which it acts (figure 1). As with drag, it is expressed as mean force per unit horizontal area of the whole bed.

The critical drag F_c acting on a spherical grain is

$$F_c = (0.52gd^3\rho' - L_c) \tan \phi' \quad [1]$$

in which ρ' is the difference in the density of the grain and the fluid, d the grain diameter, g the acceleration of gravity, and ϕ' is the angle of repose of the grain with respect to the average drag level of the fluid. In this equation the expression $0.52gd^3\rho'$ is the immersed weight of the spherical grain in dynes and L_c is the lift on the grain also in dynes.

From previous experiments (3) lift was found to be equal to about 0.85 of drag for any size of spherical roughness elements and any fluid velocity within the range of this study so that $L_c = 0.85F_c$. Therefore,

$$F_c = (0.52gd^3\rho' - 0.85F_c) \tan \phi' \quad [2]$$

which by transposing becomes

$$0.52gd^3\rho' \tan \phi' = F_c + 0.85F_c \tan \phi' \quad [3]$$

and by factoring

$$0.52gd^3\rho' \tan \phi' = F_c(1 + 0.85 \tan \phi') \quad [4]$$

so that,

$$F_c = 0.52gd^3\rho' \tan \phi'/1 + 0.85 \tan \phi' \quad [5]$$

Equation [5] indicates the critical drag required to move the top grain of diameter d. The critical drag F_c' per unit cross-sectional horizontal area occupied by the grain is equal to $F_c/0.7854d^2$ in which $0.7854d^2$ is the largest cross-sectional horizontal area occupied by the spherical grain. Then

$$F_c' = 0.52gd\rho' \tan \phi'/0.7854d^2 \ (1 + 0.85 \tan \phi') \quad [6]$$

By simplification,

$$F_c' = 0.66gd\rho' \tan \phi'/1 + 0.85 \tan \phi' \quad [7]$$

Drag and lift per unit area on the top grains is much higher than drag and lift per unit area on the whole bed because the top grains which take up most of the drag and lift occupy only a portion of the bed area. If η is the ratio of drag and lift on the whole bed to drag and lift on the top grain moved by the fluid, then equation [7] becomes

$$\tau_c = 0.66gd\rho' \tan \phi'\eta/1 + 0.85 \tan \phi' \quad [8]$$

in which τ_c is critical drag per unit horizontal area of the whole bed.

Since the airstream of a velocity required to move the top grains is not uniform, movement of the soil grains is facilitated by the maximum lift and drag impulses of turbulent flow. It is reasonable to assume that the ratio of maximum to mean drag is equal to the ratio of maximum to mean lift since from

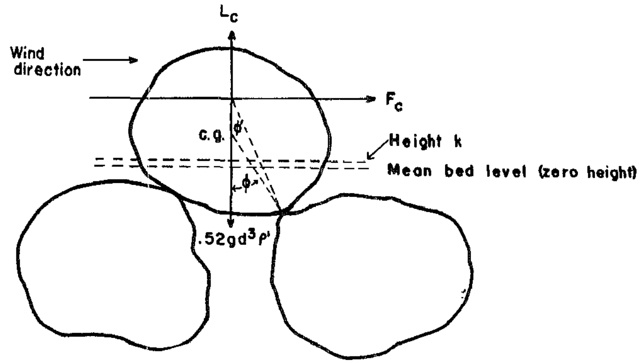

Figure 1—Forces of lift, drag, and gravity acting on a soil grain in a windstream at the threshold of movement of the grain. Net moment opposing F_c is $(0.52\ gd^3\rho' - L_c)$ tan ϕ'.

previous experiments (3) $\bar{L} = 0.85\bar{F}$. Therefore for turbulent flow, equation [8] should be modified to

$$\bar{\tau}_c = 0.66gd\rho' \tan \phi'\eta/(1 + 0.85 \tan \phi')T \quad [9]$$

in which $\bar{\tau}_c$ is the mean critical drag per unit horizontal area of the whole bed and T is the ratio of maximum to mean drag and lift on the soil grain.

EXPERIMENTAL CONDITIONS AND PROCEDURES

A wind tunnel with a test chamber 56 feet long, 3 feet wide, and 3 feet high was used to check the general validity of equation [9] and the values of constants contained in it. Grains composed of soil aggregates or gravel sieve-graded into 3.36 to 4.75 and 4.75 to 6.4 mm. in diameter comprised the floor of the tunnel. Fluid pressure and velocity measurements at the threshold of movement of the sediment grains were made 50 feet downstream and halfway between the walls. At this location the fluid boundary layer at least approached a 30-cm. depth (figure 2). Depth of the fluid boundary layer is the maximum height z to which the following well-known velocity formula near a rough surface applies (3)

$$v_z = 5.75\, V_* \log_{10} \frac{z}{k} \quad [10]$$

in which v_z is the mean velocity at height z, V_* is mean drag velocity, and k is the height above the aerodynamic surface at which the fluid velocity is zero. V_* is equal to (velocity at 30k)/8.5. Mean drag $\bar{\tau}_0$ per unit area on the whole surface was determined from V_* since $\bar{\tau}_0 = \rho V_*^2$. Mean critical drag $\bar{\tau}_c$ per unit area was determined in like manner from the mean critical or threshold drag velocity V_{*t}. Mean drag velocity and the mean drag which were barely high enough to produce a continuous movement of all aggregate sizes present in the sample of grains were considered as threshold drag velocity V_{*t} and threshold drag $\bar{\tau}_c$.

Bulk density of grains was determined by method 1 of Chepil (2).

Angle of repose ϕ with respect of force of gravity was determined for different sieve grades of the same dune sand as that used by Zingg.[3] A uniform layer of sand of each sieve grade was cemented to a smooth metal plate. A thin layer of loose grains from the same sieve grade was placed on top of the cemented layer of sand and the plate was tilted slowly until the first downward movement of individual grains became perceptible. Angle of the plate at this position with respect to the horizontal was considered the angle of repose ϕ.

Angle of repose ϕ with respect to force of gravity is the angle subtended from the center of gravity (c.g.) between the vertical and the point of repose shown in figure 1 and served merely to determine the angle of repose ϕ' with respect to the average level of critical drag F_c. A description of how ϕ' was determined from ϕ and from air velocity at various

[3]Zingg, A. W. A study of the characteristics of sand movement by wind. M.S. thesis, Dept. of Agr. Eng., Kansas State University, Manhattan, 1952.

heights will be given more conveniently in the next section of the paper.

Values of the coefficient η for sand and gravel surfaces were determined as a coefficient $\bar{\tau}_o/\bar{F}$ and $\bar{L}'/\bar{L}$ in which $\bar{\tau}_o$ and $\bar{L}'$ are the mean drag and lift per unit area on the whole bed and $\bar{F}$ and $\bar{L}$ are the mean drag and lift per unit area on the top grain or roughness element similar to the grain on a bed. Values of $\bar{F}$ obtained in a previous study (3) for hemispheres having a height-spacing ratio of 1:3 were used. This height-spacing ratio appears to predominate on loose sand and gravel surfaces (10). The mean drag $\bar{\tau}_o$ for the same size of hemispheres and fluid velocity was computed from the drag velocity V_* of the fluid in accordance with equation $\bar{\tau}_o = \rho V_*^2$. Since the ratio of lift to drag is constant, it must be assumed that η for lift is the same as for drag.

The fluid turbulence factor T_D, which is a measure of degree of fluctuation of drag at a position of the topmost grains, was determined with the strain gage assembly, amplifier, and recording oscillograph described in another publication (4) using spheres 3.2 and 6.4 mm. in diameter for this purpose. The spheres were placed at a position of the topmost grain shown in figure 1. To facilitate measurement of fluid force on the spheres, the grains below the spheres were pressed down slightly so that the spheres were in effect suspended in the fluid.

The fluid turbulence factor T_L, degree of fluctuation of lift at a position of the topmost grains, was measured indirectly as illustrated in figure 3. The method consisted of a short U shaped duct A ½ inch in diameter, one end of which was level with the top parts of the surface roughness elements and the other with the bottom. Four strain gages B were cemented together as for measuring drag and mounted at the upper end of the duct. The lead ends of the gages were insulated and cemented to the duct. The gage ends with their flat sides lying normal to the duct extended to the center of the duct. A fine rubber sheath C was cemented to the end of the duct and to the loose ends of the gages. The lower end of the duct was left open. The comparatively large size of opening did not appreciably influence the magnitude of lift pulsations because the opening was at the base of the roughness elements where the air was almost completely stagnant. The registered lift impulses were those occurring at the top of the roughness elements. Any pressure or suction on sheath C produced a strain and stress on the gages. The total magnitude

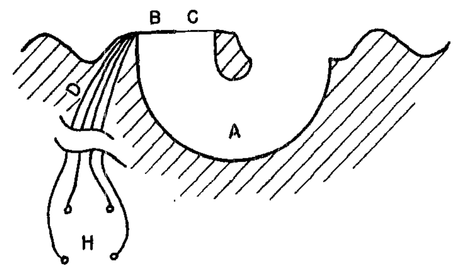

Figure 3—Cross-sectional view of apparatus for measuring lift fluctuations. Direction of wind is normal to the diagram.

of strain and stress was recorded with an amplifier and magnetic oscillograph. The gages were connected by fine wires D to a bridge H on the amplifier and in such a way as to give the greatest bridge unbalance for a given strain and stress on the gages. A pressure on one end of the duct would swing the pen of the oscillograph in one direction and pressure on the other would swing it in the opposite direction. Thus, an equal pressure or motion on both ends would give zero reading on the oscillograph. The oscillograph registered only the difference in pressure between the two levels.

The mean critical drag $\bar{\tau}_c$ for various sieve grades of dune sand reported by Zingg[4] and for various sizes of clay loam grains reported by Chepil (1) was compared with the mean critical drag $\bar{\tau}_c$ computed in accordance with equation [9] after determining the values of constants T_D, T_L, ϕ, ϕ' and η.

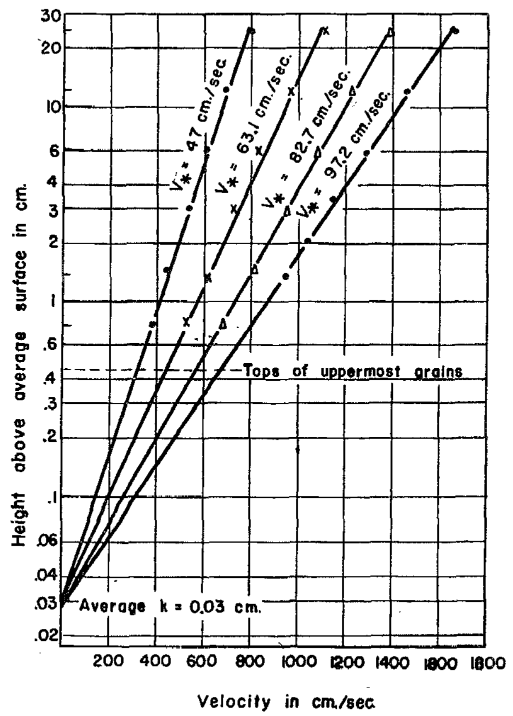

Figure 2—Wind velocity distributions over a smoothed bed of gravel 3.36 to 6.4 mm. in diameter.

RESULTS

Angle of repose with respect to center of gravity for various sizes of dune sand grains is given in table 1. This angle was found to be remarkably uniform for all sizes of grains. The average angle of repose was 33 degrees. It is shown as ϕ in figure 1.

The angle of repose ϕ' with respect to the mean level of drag for a smoothed bed of sediment grains was determined from fluid velocities at various heights shown in figure 2, assuming that drag varies as $\frac{1}{2}\rho v^2$ at fluid velocities used in these experiments. The mean level of drag was determined by multiplying the mean values of drag at various increments of height by area of grain normal to wind at corresponding height increments, adding up the products, and determining graphically the exact height at which one-half the cumulative total occurs (table 2). The data indicate that for spherical grains 0.64 cm. in diameter one-half of drag pressure F_c occurs at about 0.265 cm. above the mean bed level. This height therefore is approximately the mean level at which drag F_c acts. It is shown in figure 2 that the tops of these grains occur at 0.45 cm. above the mean bed level. Therefore the mean level at which F_c acts occurs at $0.45 - 0.265 = 0.185$ cm. below the tops of the grains. Expressed in dimensionless terms, this is equal to $0.185/0.64 = 0.29$, of grain diameter below the tops of grains. Approximately the same relative values for mean level of drag were obtained using

[4]Ibid.

Table 1—Angle of repose ϕ for dune sand (averages of duplicated tests).

Sieve grade	Angle of repose, ϕ
mm.	degrees
0.1 -0.15	33
0.15-0.25	33
0.25-0.30	35
0.30-0.42	34
0.42-0.59	31
0.59-0.84	32
0.84-1.19	35
Average	33

various drag velocities and sizes of grains. This level is indicated in figure 1 by horizontal line F_c. The angle ϕ' subtended from this line between the point of repose and the vertical passing through the center of gravity is the angle of repose with respect to the average drag level of the wind. It was found to be on the average about 24 degrees. Therefore, tan ϕ' is equal to 0.445.

The coefficient η was found to have a mean value of 0.21 for hemispherical roughness elements such as the soil grains spaced in a hexagonal pattern 3 diameters apart on top of the bed (table 3). If the drag and lift were all taken up by the top grains moved by wind, the value of η for this condition would be equal to the ratio of horizontal area occupied by the top grains moved by the fluid per unit area of the bed. Accordingly for spherical grains spaced 3 diameters apart η would be equal to about 0.073. Therefore, it is evident that nearly two-thirds of the drag and lift were taken up by grains other than those moved by wind.

Figure 4 indicates the magnitude of pressure fluctuations at the position of the topmost grains on the bed. The mean pressure of lift and drag obtained from these records agreed quite well with the mean obtained with taps on the hemispherical roughness elements connected to an alcohol manometer (3). The ratio of maximum to mean pressure appears to be about 2.5 to 1. However, this is merely a visual indication. Analyses of oscillograms such as those of figure 4 indicated that pressure of both lift and drag at a level of the topmost grains is statistically distributed according to a somewhat skewed normal error law (4). Hence, from a statistical standpoint, the maximum pressure of lift or drag has no definite limit and therefore the ratio of maximum to mean cannot be given. The standard deviation, however, completely describes the spread of pressure of lift and drag around the mean. Analyses showed that the standard deviation σ of the pressure distributions varies directly with the drag velocity of the wind. However, the ratio of mean pressure to standard deviation appears to be nearly constant at the position of topmost grains on a bed and may be expressed by an equation $\sigma = c\bar{P}$ in which $\bar{P}$ is mean pressure of lift or drag and c is a constant which was found to have a mean value of 0.49 (table 4). Its value appeared to be almost constant for beds of grains or spherical mounds ranging from about 0.16 to 6.35 cm. in diameter. The variation in individual values of c was apparently due to error of measurement. The measurements were for time periods of approximately 2.5 seconds. Each measurement was based on 480 consecutive pressure fluctuations of the oscillogram. A much more accurate evaluation of pressure fluctuations and more constant values of c would have been obtained for time periods of at least 15 seconds, but faced with a large number of analyses that had to be made for various wind velocities and sizes of grains, this period was prohibitive for use in all cases.

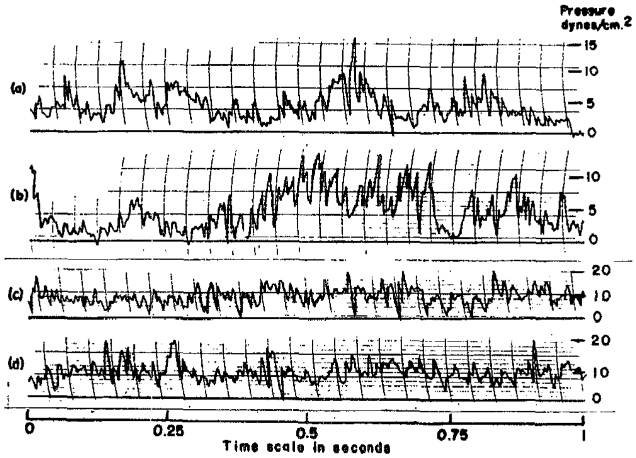

Figure 4—Oscillograph records of pressure of drag and lift on spherical gravel 3.2 and 6.4 mm. in diameter at bed level for a drag velocity of 47 cm. per second: (a) Drag on 3.2 mm. gravel, (b) Drag on 6.4 mm. gravel, (c) Pressure difference between top and bottom of 3.2 mm. gravel, (d) Pressure difference between top and bottom of 6.4 mm. gravel.

It is evident from the nature of pressure distributions that turbulence factors T_L and T_D of lift and drag, respectively, cannot be given in terms of ratio of absolute maximum to mean pressure because no distinct absolute maximum exists (4). However, nearly all or 99.73% of the pressure range is included within $\bar{P} \pm 3\sigma$ in which $\bar{P}$ is the mean pressure and σ is the standard deviation for the frequency distribution of the pressure of lift or drag. The turbulence factors therefore were taken as $(\bar{P} + 3\sigma)/\bar{P}$ which assumes that the "maximum" pressure is $\bar{P} + 3\sigma$. On the basis of this assumption the turbulence factors T_L and T_D were found to be approximately equal and to have an average value of 2.47. Therefore, the turbulence factors T_L and T_D can be considered as a single factor T as assumed in equation [9].

Negative pressure of lift and drag, as indicated by frequency of occurrence of zero pressure f_o' and f_o, respectively, occurred on the average about 1% of the time (table 4).

Oscillograms of figure 4 show that the smallest-scale cycles of pressure, although irregular both as to magnitude and duration of occurrence, have a period of about 1/80 to 1/120 second. Thus, the negative pressure at the level of the topmost grains on the bed occurs approximately about once each second. Duration of these primary periods

Table 2—Determination of level at which pressure F_c acts on the topmost spherical grains 0.64 cm. in diameter. (From figures 1 and 2, $V_* = 47$ cm./sec.).*

Height above average bed level	Mean velocity v	Mean drag τ	Area of grain normal to wind A	Total drag τA
cm.	cm./sec.	dynes/cm.²	cm.²	dynes
0 -0.05	0	0	0.0301	0
0.05-0.1	100	6	0.0315	0.189
0.1 -0.15	162	16	0.0320	0.512
0.15-0.2	202	24	0.0315	0.756
0.2 -0.25	232	32	0.0305	0.977
0.25-0.3	253	38	0.0285	1.085
0.3 -0.35	273	45	0.0262	1.180
0.35-0.4	292	51	0.0215	1.099
0.4 -0.45	308	57	0.0115	0.657
Total				6.455
Half of total				3.228

* One-half of drag F_c (one-half of cumulative total τA) against the topmost grains occurs below and one-half above the height of 0.265 cm. above the average bed level.

Table 3—Ratio $\eta = \bar{\tau}_0/\bar{F}$ on hemispheres spaced 3 diameters apart in a hexagonal pattern.

Height of hemisphere	V_*	Mean drag $\bar{\tau}_0$ on whole surface computed from V_*	Mean drag $\bar{F}$ on hemisphere	Ratio $\bar{\tau}_0 : \bar{F}$
cm.	cm./sec.	dynes/cm.²	dynes/cm.²	
1.27	57	3.9	16	0.24
1.27	79	7.5	28	0.27
1.27	111	14.8	55	0.27
1.27	138	23.0	91	0.25
2.54	68	5.5	27	0.20
2.54	91	10.5	48	0.22
2.54	128	19.6	102	0.19
2.54	159	30.4	156	0.20
5.08	76	7.0	41	0.17
5.08	109	14.4	96	0.15
5.08	146	25.6	142	0.18
5.08	188	42.7	217	0.20
Average				0.21

Table 4—Relation of mean pressure and standard deviation of pressure of lift and drag on sand and gravel surfaces of various magnitudes of roughness to constants c, f_o, f_o', T_L, and T_D.

Surface	Drag velocity of wind	Pressure difference between top and bottom of topmost grain or mound						Drag on sphere at mean surface					
		Diameter of grain or mound	$\bar{P}_L$	σ	c	f_o'	T_L	Diameter of sphere	$\bar{P}_D$	σ	c	f_o	T_D
	cm./sec.	cm.	dynes/cm.^{2}	dynes/cm.^{2}		%		cm.	dynes/cm.^{2}	dynes/cm.^{2}		%	
	40.0	0.16	11.4	6.4	0.66	0.4	2.69	0.16		Force too small to measure			
Wet sand	54.7	0.16	16.9	8.4	0.50	1.7	2.48	0.16	7.4	4.4	0.59	1.9	2.78
0.84 to 1.68 mm.	73.5	0.16	24.7	9.5	0.38	0.4	2.15	0.16	10.4	6.1	0.59	0	2.76
in diameter	99.2	0.16	28.8	11.7	0.41	0	2.22	0.16	10.8	6.2	0.57	0	2.75
	Aver.	0.16	-	-	0.46	0.6	2.38	0.16	-	-	0.58	0.6	2.76
Gravel	40.0	0.32	9.7	6.4	0.66	1.8	2.98	0.32		Force too small to measure			
2 to 3.36 mm.	56.5	0.32	18.5	9.8	0.53	0.9	2.59	0.32	9.4	5.0	0.53	1.0	2.59
in diameter	72.7	0.32	35.2	17.0	0.48	2.5	2.47	0.32	20.2	10.6	0.52	0	2.57
	91.3	0.32	40.8	19.9	0.49	2.7	2.46	0.32	29.6	17.1	0.58	1.5	2.74
	Aver.	0.32	-	-	0.54	2.0	2.62	0.32	-	-	0.54	0.8	2.63
Gravel	47.0	0.64	20.1	9.9	0.49	0.8	2.48	0.64	12.8	5.8	0.45	1.4	2.36
3.36 to 6.4 mm.	63.0	0.64	36.2	15.9	0.44	1.0	2.32	0.64	22.4	11.1	0.49	1.6	2.48
in diameter	82.7	0.64	63.0	24.9	0.40	0.6	2.18	0.64	33.0	14.3	0.43	1.3	2.30
	97.2	0.64	85.0	34.2	0.40	1.2	2.21	0.64	52.8	24.6	0.47	1.0	2.39
	Aver.	0.64	-	-	0.43	0.9	2.30	0.64	-	-	0.46	1.3	2.38
Spherical gravel	72.9	6.35	37.5	15.1	0.40	0.5	2.21	0.64	22.0	11.0	0.50	1.5	2.49
mounds 6.35 cm.	111.8	6.35	74.0	31.4	0.42	0.5	2.27	0.64	32.5	16.6	0.51	0.7	2.53
in diameter, 3	142.4	6.35	145.0	67.5	0.47	0.4	2.39	0.64	58.0	30.2	0.52	0.1	2.56
diameters apart	170.0	6.35	295.0	122.4	0.41	0.7	2.25	0.64	100.0	45.2	0.45	0.5	2.36
	Aver.	6.35	-	-	0.42	0.5	2.28	0.64	-	-	0.50	0.7	2.48

varied little with all wind velocities used. Pressures equal to or greater than $\bar{P} + 3\sigma$ for lift and drag occurred 2 to 3 times per second, depending somewhat on drag velocity of the fluid.

There was some evidence that upward and forward impulses of turbulent flow were stronger than downward and backward at the position of the topmost grains on the bed. These impulses are indicated by upward swings of the oscillograms (figure 4). For all practical purposes the bottoms of the downward swings, which may be designated by $\bar{P} - 3\sigma$, occurred at about zero pressure. If the swings were perfectly symmetrical about the mean and $\bar{P} - 3\sigma$ occurred at zero pressure, the values of turbulence factors T_L and T_D should have been about 2. Actual values substantially greater than 2 must therefore be attributed principally to greater upward and forward than downward and backward pressure impulses at the position of the topmost grains.

APPLICATION OF RESULTS

In accordance with data obtained in these experiments, the following values for the different constants of equation [9] were taken: $T = 2.5$, $\tan \phi' = 0.445$, and $\eta = 0.21$. All grains were assumed to be spherical. Computations were based on the smallest grains of each sieve grade tested. The smallest grains were chosen because it is known that movement of the larger grains is sustained by impacts of the smaller ones (1).

For dune sand the actual threshold drag determined by Zingg[5] and drag computed in accordance with equation [9] are shown in table 5. The computed threshold drag was somewhat higher than the actual drag for the values of the constants used. An increase in T from 2.5 to 3 would bring the computed threshold drag in general agreement with the actual threshold drag. The value of 3 for T is not unreasonable since it is known from the information reported in this paper that pressure impulses which give T a value equal to or greater than 2.5 occur 2 or 3 times per second. A slight decrease in $\tan \phi'$ and η would also bring the two sets of data in general agreement with each other.

Equation [9] also was checked against the data by Chepil (1) for various sieve grades of clay loam soil. This comparison is given in table 6 and shows that computed threshold drag is somewhat higher than the actual threshold drag. Again, a near-perfect agreement would be obtained if T were increased to 3 or if $\tan \phi'$ and η were decreased slightly.

[5]Ibid.

Table 5—Concordance of computed with actual threshold drag for sand grains of uniform density.*

Actual grain diameter	Minimum grain diameter d	Immersed grain density ρ'	Computed threshold drag, $\bar{\tau}_c$	Actual threshold drag, $\bar{\tau}_c$
mm.	cm.		dynes/cm.^{2}	dynes/cm.^{2}
0.15 - 0.25	0.015	2.65	0.69	0.59
0.25 - 0.30	0.025	2.65	1.16	0.85
0.30 - 0.42	0.030	2.65	1.39	1.11
0.42 - 0.59	0.042	2.65	1.95	1.60
0.59 - 0.84	0.059	2.65	2.74	2.32

* Actual threshold drag values are from data by Zingg (10).

Table 6—Concordance of computed with actual threshold drag for soil grains of different densities.*

Actual grain diameter	Minimum grain diameter	Immersed grain density ρ'	Computed threshold drag, $\bar{\tau}_c$	Actual threshold drag, $\bar{\tau}_c$
mm.	cm.		dynes/cm.^{2}	dynes/cm.^{2}
0.1 -0.15	0.01	2.09	0.37	0.27
0.15-0.18	0.015	1.96	0.51	0.35
0.18-0.25	0.018	1.94	0.61	0.48
0.25-0.42	0.025	1.91	0.84	0.69
0.42-0.59	0.042	1.91	1.40	1.08
0.59-0.84	0.059	1.80	1.86	1.51
0.84-1.19	0.084	1.78	2.62	2.33
1.19-2	0.119	1.74	3.63	3.59
2 -3	0.20	1.65	5.78	5.15

* Immersed density and actual threshold drag values are from data by Chepil (1).

Table 7—Concordance of computed with actual soil aggregates of different densities.

Actual aggregate diameter	Minimum aggregate diameter	Immersed aggregate density ρ'	Computed threshold drag, $\bar{\tau}_c$	Actual threshold drag, $\bar{\tau}_c$
mm.	cm.		dynes/cm.^{2}	dynes/cm.^{2}
3.36-4.75	0.336	1.52	8.94	10.3
4.75-6.4	0.475	1.55	12.89	14.0

The actual threshold drag and the threshold drag computed in accordance with equation [9] for extremely large soil grains (aggregates) of low density are given in table 7. The values of computed threshold drag in this case are somewhat smaller than the actual. It is known that the grains were considerably more angular than those of tables 5 and 6. The computed values of threshold drag would be somewhat greater than the actual if the grains were assumed to be cubical instead of spherical. Actually they were irregular in shape and on the average were probably equivalent to those between a sphere and a cube.

The data presented in tables 5, 6, and 7 seem to confirm the general validity of equation [9] and the approximate values of the constants which it embodies.

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