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# A CONSERVATION DEFINITION OF EROSION TOLERANCE

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In the United States a widespread belief in the necessity for control of soil erosion has led to the support of vigorous measures by State and Federal agencies engaged in various phases of such control. In addition, millions of farmers, working individually and in organized groups, have put into practice the conservation ideas developed through scientific research and experience. All this effort has resulted, particularly during the last 30 years, in reducing the rate of loss of the soil resource.

Although there is general agreement that any erosion represents a potential loss of an essential resource, there is no generally accepted standard for determining precisely what kind of erosion or how much is tolerable or permissible. Some support the view that efforts should be directed toward the elimination of all accelerated soil erosion and of some geologic erosion, while many others question this belief and even suggest the possibility that in some cases a certain amount of accelerated erosion is beneficial.

This paper formulates an expression of erosion tolerance and related concepts to serve as a guide for future conservation research and practice.

## BASIC ASSUMPTIONS AND MOTIVATION

Before formulating the definition in mathematical symbolism, it is necessary to specify what assumptions are being made, that is to list the necessary *a priori* conditions such a definition must satisfy along with a clarification of the use of certain terminology. The definition of erosion tolerance must:

1. Provide for the *permanent* preservation or improvement of the soil as a resource.
2. Be adaptable to the erosion (wearing away) and renewal rates of any soil characteristic [erosion, which is the removal or depletion of some soil characteristic (available depth, texture, plant nutrients, etc.) is accompanied by renewal. This is the addition to any characteristic. The difference between the renewal rate and the erosion rate is the net rate of change.]
3. Be a function of position (or location), since at any two points on the earth's surface the erosion and renewal rates will not necessarily be identical.
4. Be applicable regardless of the cause of erosion or renewal.
5. Based on the assumption that if any soil property (depth, for example) is available in excess of present or predictable future requirements, it is tolerable to use up the excess.

In order to make more meaningful the definition that will be given, it might be useful to present some reasoning that underlies the mathematical expression. Accordingly, consider the interval of time $t$ from the present time $t_0$ to some future time $T$, and let $\Delta t$ be a sub-interval of time in the interval from $t_0$ to $T$. Let $P$ be a point on the earth's surface, and let $t^*$ be an instant of time in $\Delta t$. Let $t^*$ be further restricted so that the product of $\Delta t$ with the time rate of change of some soil characteristic at $P$, at time $t^*$, gives the total change at $P$ during the interval of time $\Delta t$.

Now let the interval of time from the present $t_0$ to time $T$ be divided into some finite number $n$ of sub-intervals. Suppose that for each $k = 1, \cdots, n$, there is in the time interval from $t_{k-1}$ to $t_k$ an instant of time $t_k^*$ such that the product of $(t_k - t_{k-1})$ with the instantaneous time rate of net change of some soil characteristic at point $P$, at instant $t_k^*$, represents the total net change in this characteristic at $P$ during the time period from $t_{k-1}$ to $t_k$. Then the sum of these products as $k$ ranges over the values from 1 to $n$ will represent the total change in this characteristic which will take place at $P$ from the present until time $T$. The conservation definition of

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erosion tolerance must provide a lower bound which the difference between the present measure of the soil characteristic and the algebraic sum of these products must exceed, *no matter how great the time interval from* $t_0$ *to* $T$ (that is, present measure − total change ≧ lower bound).

### THE DEFINITION

Let it now be assumed that at point $(x, y)$ the net change rate of the particular soil characteristic under study is a function $F(x, y, t)$ which depends on time $t$ in some manner so that if $T$ is any particular positive number, the integral

$$\int_{t_0}^{T} F(x, y, t)\, dt \qquad (1)$$

defines a function of $x$ and $y$ only. If function $F(x, y, t)$ is a continuous function of $t$, then the integral will certainly define a function of $x$ and $y$. Furthermore, this function of $x$ and $y$ will then coincide with the function in the preceding paragraph which gives the total change of the measure of a soil characteristic at point $P$, from the present time ($t_0$) until some future time ($T$). For the purposes of this paper it is necessary to assume only that the integral (*1*) defines a function of position or location. It is unnecessary, perhaps undesirable, to make the stronger assumption that $F(x, y, t)$ is a continuous function of $t$.

According to the assumption that conservation is the *permanent* preservation or improvement of a certain productive capacity, the definition of erosion tolerance must be stated in terms not of finite time $T$ but rather in the results obtained by integrating over an infinite time interval. The following definition is thus formulated.

Definition of net change tolerance: $F(x, y, t)$ is a function of position (location) and time which represents at the end of $t$ units of time the time rate of net change of the measure at point $(x, y)$ of some measurable soil property. Let $M(x, y)$ represent the minimum allowable value at $(x, y)$ of the measure of this property. Let $I(x, y)$ be the position function which gives the value of this measure at the present time ($t_0$). Then the function $F(x, y, t)$ represents at point $(x, y)$ a *net change tolerance* if

$$I(x, y) - \int_{t_0}^{\infty} F(x, y, t)\, dt \geqq M(x, y) \qquad (2)$$

Now select two functions of position and time $E(x, y, t)$ and $R(x, y, t)$ which satisfy the conditions:

1. $E(x, y, t) \geqq 0$
2. $R(x, y, t) \geqq 0$
3. $E(x, y, t) - R(x, y, t) = F(x, y, t)$.

Obviously there are infinitely many ways in which a given function $F(x, y, t)$ may be represented as the difference of two non-negative functions. From the standpoint of soil conservation it is just as obvious that the interesting representations will be ones in which $E(x, y, t)$ represents the erosion rate of some measurable soil property and $R(x, y, t)$ the renewal[2] rate of that property. Then the function $E(x, y, t)$ will be the result of such influences as wind, water, plant action, and man-made changes. These same influences, together with soil formation from below, will affect the choice of function $R(x, y, t)$.

Henceforth, much of the discussion concerns the change in some soil property at an arbitrary but fixed point $(x', y')$. At this fixed point, equation (*2*) may be expressed as

$$I(x', y') - \int_{t_0}^{\infty} [E(x', y', t) - R(x', y', t)]\, dt \geqq M(x', y') \qquad (3)$$

Just as $F(x', y', t)$ is by definition the net change tolerance at point $(x', y')$ the function $E(x', y', t)$ in equation (*3*) is the erosion tolerance function at point $(x', y')$.

### PROPERTIES OF NET CHANGE TOLERANCE

Equation (*3*) provides (theoretically at least) a means for the determination of a function which represents at a particular point the instantaneous net rate of change of some soil characteristic. Since the function to be determined from equation (*2*) appears as the integrand, there is an infinite number of functions which could satisfy the equation. The actual determination, then, of a particular function can be made to depend on other influences. This will be discussed later in this paper.

The definition, equation (*2*), is general enough to include both selective and non-selective

[2] The term renewal as used in this paper includes all "natural" or "artificial" additions to the soil by means of deposition or weathering.

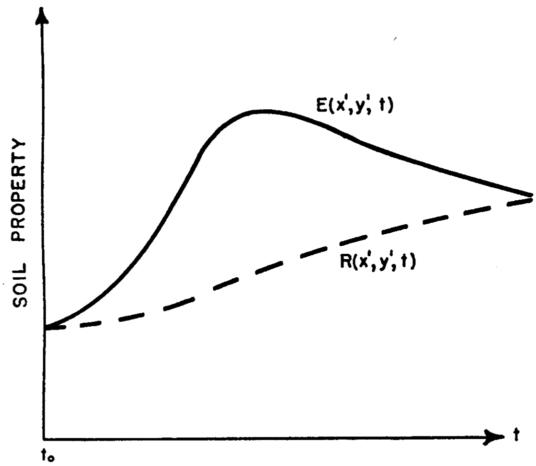


FIG. 1. Graphs of possible functions $E(x', y', t)$ and $R(x', y', t)$ when $t_0$ is time of initial cultivation of a deep virgin soil lying over easily weathered rock.

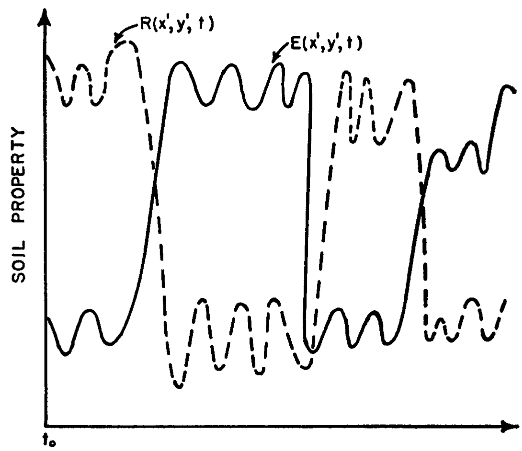


FIG. 2. Illustration of possible relations between functions for erosion $[E(x', y', t)]$ and soil renewal $[R(x', y', t)]$ with rotation of crops in a wind-erosion region.

erosion and renewal. If non-selective change is being considered, then $M(x', y')$ may represent the minimum permissible depth of usable soil, measured in linear units, say feet. On the other hand, the definition may also be applied to the rate of change of some measurable physical or chemical property of the soil. The only requirement is that $M(x', y')$ be expressible in some set of units at the arbitrary point $(x', y')$.

### THE CHOICE OF A FUNCTION $E$

It cannot be emphasized too strongly that equation (*3*) allows a very wide choice in the selection of a function $E(x', y', t)$. One possible solution is illustrated in figure 1. The function $E(x', y', t)$ increases rapidly to a maximum value and then decreases slowly thereafter, while $R(x', y', t)$ is small near $t_0$ and increases with time until it equals $E(x', y', t)$. This choice of functions $E(x', y', t)$ and $R(x', y', t)$ expresses the behavior which might be expected when $t_0$ represents the time of initial cultivation of a deep virgin soil lying over easily weathered rock. The representation of $R(x', y', t)$ as an increasing function displays the increased rock-weathering rate in response to reduced depth of soil material as erosion progresses.

An entirely different case is shown in figure 2 where $E(x', y', t)$ and $R(x', y', t)$ both oscillate, essentially in opposition. When the one function is large, the other tends to be small. This represents a case which might occur in a wind erosion region with a rotation of alternately high residue crops and clean plowing. During periods of clean plowing, removal would be rapid, though variable with the weather, whereas renewal would be slow, consisting mainly of underlying rock weathering. With high residue crops, however, $E(x', y', t)$ would be small and $R(x', y', t)$ would be large. The increase in $R(x', y', t)$ would be the result of deposition and trapping of soil material from the air.

In order to determine which of the multitude of available functions will be suitable to a particular case, it will be necessary to apply additional criteria to $E(x', y', t)$ and $R(x', y', t)$. These criteria may result from economic considerations and personal or public preference.

### THE DETERMINATION OF $M$

The function $M(x', y', t)$ will reflect the conservationist's knowledge of the long-range requirements for a particular essential soil property such as depth of favorable material. It should be large enough to include a margin for possible error. The error may be due to imperfect knowledge since provision is being made for permanence of the given soil property.

### EROSION TOLERANCE OVER A REGION

In the foregoing, erosion tolerance has been considered only at a single point. This can be expanded to any given region, from a garden plot to a continent, if the necessary information is obtainable. The functions $I(x, y)$ and $M(x, y)$ must be determined for every point $(x, y)$ of the region. If the region under discussion is denoted by $A$, and if the functions $I$, $E$, $R$, and $M$ are assumed to be integrable over region $A$, then equation (*3*) leads to

$$\int_A \int I(x, y)\, dA - \int_A \int \left\{ \int_{t_0}^{\infty} [E(x, y, t) - R(x, y, t)]\, dt \right\} dA \geqq \int_A \int M(x, y)\, dA \quad (4)$$

The use of equation (*4*) would permit the introduction of average values. For example, instead of using a (variable) function for $M(x, y)$, a constant average value, say $M^*$, such that

$$M^*A \geqq \int_A \int M(x, y)\, dA \quad (5)$$

could be employed.

## SUMMARY

A mathematical expression has been developed that appears to express the concepts involved in such terms as "soil erosion tolerance" ("permissible soil loss") and "soil renewal."

This expression provides for the use of soil property reserves and the permanent protection or improvement of soil resources in accordance with measurable standards. Net change from present condition is stated by a definite integral involving soil erosion and soil renewal (or addition) rates with time. The basic equation applies to any point on the land surface but can be expanded over a region. Both wind and water erosion are considered, as well as selective and nonselective erosion, and controlled or uncontrolled influences.

The equation clarifies concepts of erosion and renewal, points out prime needs for experimentation, and suggests feasibility of certain types of solutions. Information needed for solution is: (*a*) specific inventory of present soil resources; (*b*) expression of essential soil-property requirements for the future; (*c*) data on erosion (or wearing away) of soil properties with time; and (*d*) data on renewal (or additions) to soil properties with time. Choice of a particular solution will depend upon economic influences, personal preference, and public policy.