A Method for Determining Field Wind Erosion Rates from Wind-Tunnel-Derived Functions

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ABSTRACT

THE mass flow rate equation for a convex-shaped field surface subjected to wind erosion is derived by the application of the steady state continuity equation. It is assumed and justified that the soil flow can be idealized as a flux.

The resultant equation, predicted on the availability of line-intensity functions (q) developed from wind tunnel studies, is the line integral of the q's around the perimeter of the field at the saltation height. The shape of the field is limited at present to only convex shapes. Fortunately, a typical agricultural field is rectangular.

The assumptions implied by this method are stressed and two examples are presented that deal with nonhomogeneous surfaces and both erodible and nonerodible boundaries.

INTRODUCTION

In the development of any equation or method for computing wind erosion soil loss from a field, one is immediately faced with the enormity of the task. This is due primarily to one's ability to see more details of the erosion process than can be dealt with. Consequently, the use of a simplifying model is just as desirable here as in any other scientific field. Early researchers (Bagnold, 1941; Chepil, 1959) surely recognized this problem, and although they may not have specified their simplifying assumptions, these were implied and were necessary to handle the prediction problem.

One important assumption is that a wind tunnel can simulate the wind erosion process adequately so that soil loss can be related to many of the important variables that influence it. Obviously, the surface of the wind tunnel does not represent the total field surface that is of interest. Furthermore, due to the small size of the soil sample and the lack of soil abrasion, the time duration is rather short, i.e., minutes as compared to hours on a field. As a consequence of the small sample, there has been a difference in the measured dependent variable between tunnel and field. As Chepil (1959) indicated, the mass, m, was the tunnel variable whereas the variable in the field was q. (All symbols are identified in Table 1.) We see, therefore, that only a partial or incomplete model of the field erosion process can be formulated from tunnel data and, at most, the data would be representative of a small line segment of a large field.

Chepil realized the limitations of his initial tunnel-

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TABLE 1. NOTATION. M, L, AND T AS DIMENSIONS REFER TO MASS, LENGTH, AND TIME.

Symbol	Definition and dimension
A	area of a surface, L^2
C	the perimeter of S_1 or S_2, L
E	potential average annual soil loss, $M\ L^{-2}\ T^{-1}$
$\underline{f}$	soil flux vector in r, u, z coordinates, $M\ L^{-2}\ T^{-1}$
$\bar{\underline{f}}$	soil flux vector in R, u, z coordinates, $M\ L^{-2}\ T^{-1}$
h	distance from soil surface to top of the control volume, see Fig. 1. This also may be considered the saltation height, L.
I	soil erodibility, $M\ L^{-2}\ T^{-1}$
J	The set of surface conditions indicated in equation [2] or the i-th region for application Case I and II.
K	soil ridge roughness, dimensionless
ℓ	length. The longer dimension of a rectangular field, specified along the y axis, L.
M	Soil moisture, dimensions unknown
m	the soil mass that has flowed through a specified surface for a given interval of time, see Table 2, M
m″	soil surface density, see Table 2, $M\ L^{-2}$
$\dot{m}$	the soil mass flow rate through a specified surface, see Table 2, $M\ T^{-1}$
q	line intensity, the soil flow rate per unit width. When subscripted, it implies a specific direction of integration of a normal flux vector. See for example equations [5] and [6]; $M\ L^{-1}\ T^{-1}$.
$\bar{q}$	same as q but with respect to the R, z axis, $M\ L^{-1}\ T^{-1}$
Q	A general functional form, unknown.
r	distance along the r axis, L
R	distance along the R axis, L
S_i	surface area of the i-th surface of the control volume, L^2
s	arc length of perimeter C, L
T	a time interval, T
t	time, T
U	windspeed, $L\ T^{-1}$
u	distance along the u axis, L
V	equivalent quantity of vegetative cover, M/L^2, or volume of the control volume, L^3
w	width. The narrower dimension of a rectangular field, specified along the x axis, L.
x	distance along the x axis, L
y	distance along the y axis, L
z	distance along the z axis, L
α	field angle, the angle of the positive y axis relative to north, clockwise positive, see Fig. 2, dimensionless
β	wind angle, the angle of the wind relative to the positive x axis, counterclockwise positive, see Fig. 2, dimensionless
β_1	defined by equation [47], dimensionless
γ	The path of intergration around the perimeter of an R, z plane of the control volume of Fig. 1.
Δ	difference operator, dimensionless
ρ	soil density, $M\ L^{-3}$
θ	wind angle, the angle of the wind vector relative to north, clockwise positive, see Fig. 2, dimensionless
π	3.14159 . . . , dimensionless
ψ	The inverse function of $q_r(r, z)$
Subscripts	
i	index, 1, 2, 3 . . . various surfaces and or arc lengths
n	normal component, or upper limit of an index
R	R component
r	r component
z	z component
u	u component
Superscripts and other symbols	
$\sim$	the variable may change in time
∇	Del, see equation [21]
$\langle\rangle_{\Delta x}$	an average of the function within the brackets with respect to an interval that is shown here as Δx. If the interval is unambiguous, it is omitted.
$\triangleq$	defined
$\wedge$	see equations [43] and [44]

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derived wind erosion equations to predict the field soil loss over an extended period of time. One of his early equations (Chepil, 1959), which was based on wind tunnel experiments and some field experiments (Chepil, 1957), predicted what he called a relative soil loss. In order to overcome this nondimensional soil loss representation, he performed soil loss measurements on 69 fields for 3 yr near Garden City, Kansas (Chepil, 1960). In that paper he correlated the result of his measured soil loss density, m″, with the dimensionless predicted values, and he developed a functional relationship which, in conjunction with his dimensionless tunnel equation, allowed him to predict field losses. As pointed out by Cole et al. (1982), this new equation represented the equivalent of a time and space average of a surface soil loss flux function. Chepil later extended his "3-yr equation" to a long-time average (40 yr) by using a climatic factor and a multiplicative factor of 1/3. The latter was to compensate for the fact that the 3-yr period had a higher than average climatic factor and, consequently, it was assumed higher than average soil loss fluxes (Chepil et al., 1962).

From the preceding discussion, we can see the difficulties that Chepil experienced in order to accomplish the conversion of the wind-tunnel-derived function for use in field predictions. Furthermore, it illustrates the problems that must be faced at present to apply a wind-tunnel-derived equation to a field situation.

The method presented here considers a flux equation or its first integral, q, which is derived from wind tunnel data. This function is then integrated across the field (and in time) to produce the soil loss mass. The integration is based on the conservation of mass principle and does not involve the concept of relative field erodibility that was used previously. The continuity equation, while not used previously for wind erosion, has been used for water erosion (Foster and Meyer, 1972; Scoging, 1978).

The research reported here is part of a program whose objective is to develop a method for predicting soil loss from a field for a single windstorm. The main difference between this and the existing wind erosion equation is the time duration over which the erosion process is averaged, i.e., 6 hours vs. 40 years. The basic conversion problem, i.e., integration, remains the same.

In order to view the complete integration process and to see where this particular study fits in, it is convenient to formulate the surface soil loss process as the time and space average of the normal component of the surface flux vector, i.e.,

$$E = \frac{1}{AT} \int_T \int_A \bar{f}_z \, dA \, dt \qquad [1]$$

where

$$\bar{f}_z = \bar{f}_z\,(J(t), R, u, 0) \text{ and}$$

$$J \triangleq \{\tilde{I}, \tilde{K}, \tilde{V}, \tilde{U}, \tilde{M}, ?\} \qquad [2]$$

(The use of the tilde above the independent variables indicates that they may vary in time.) The independent varible, E, of equation [1] is dimensionally identical to the E of the wind erosion equation (Woodruff and Siddoway, 1965). This is to facilitate any possible future comparisons of the two computational methods.

From equation [2], two tasks are evident. First, the development of the flux function from wind tunnel and perhaps limited size field experiments. Second, the description of how the independent variables vary in time. Equation [1] illustrates the third task, i.e., conversion of $\bar{f}$ by integration over a specified area and in time. It is the method and problems associated with the spacial integration that are discussed here, along with the required coordinate systems and the model assumptions. It is assumed that equation [2] would be available to allow the integration. As shown later, this is a reasonable assumption.

ANALYSIS

The loss of soil, no matter how it is quantified, i.e., soil loss (m), soil loss surface density (m″), soil loss flow rate ($\dot{m}$), soil loss line intensity (q), or soil loss flux (f) (see Table 2 for relationships between these forms) is basically a flow problem analogous to the fluid flow problems of fluid mechanics. In fact, what is apparent is that we have essentially two interacting flows, i.e., a multiphase flow. The concepts of mass, energy, and momentum conservation are therefore applicable.

Using these basic principles implies that the airborne soil particles behave as a fluid, i.e., they are a continuum. Crowe and Smoot (1979) dealt with this problem when developing the conservation equations for

TABLE 2. SOIL FLOW TERMS, DEFINED FOR A u,z PLANE OF AREA A IN THE R, u,z COORDINATE SYSTEM.*

Name	Functional form	Dimensions	Independent variables
Normal component of the soil flux vector $\bar{f}$	$\bar{f}_R = \bar{f}_R\,(R, u, z, J(t))$	$M\,L^{-2}\,T^{-1}$	A point in time and space
Soil mass	$m = \int_0^T \int\int_A \bar{f}_R \, du\, dz\, dt$	M	Area and time interval
Soil flow rate	$\dot{m} = \int\int_A \bar{f}_R \, du\, dz$	$M\,T^{-1}$	Area
Soil surface density	$m'' = \int_0^T \bar{f}_R \, dt$	$M\,L^{-2}$	Time interval
Soil line intensities	$\bar{q}_R = \int_0^z \bar{f}_R \, dz,$	$M\,L^{-1}\,T^{-1}$	A length interval
	$\bar{q}_z = \int_0^R \bar{f}_z \, dR$	$M\,L^{-1}\,T^{-1}$	A length interval

* If the soil flow constitutes a loss from a surface, then the word loss may be appended to the word soil, if such clarification is needed.

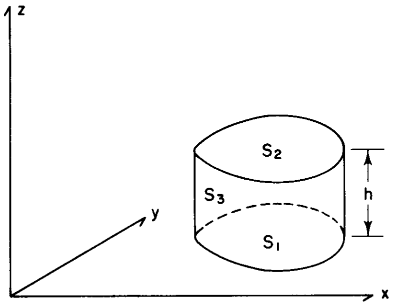

Fig. 1—A control volume for a general convex cylindrical shape showing the three sides and the height.

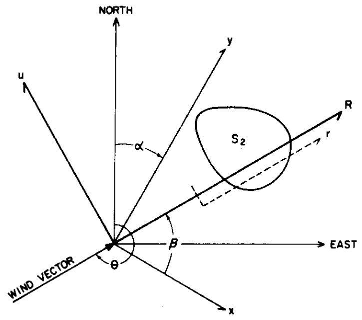

Fig. 2—A plan view of the control volume with its four required coordinate systems at z=h and the associated angles.

gas-particle mixtures related to coal combustion.

The continuum assumption, while generally not applicable for finite size particles such as soil, is felt to be reasonable since we are not interested in the flow of particles through small regions relative to the size of the particles. In the wind tunnel, the volume of interest is about 0.33 m × 2 m × 1 m, and in the field even this volume approximates a point. As will be noted later, the wind tunnel data approximates a line segment (i.e., Δq) rather than a point (i.e., f) and, as noted in Table 2, q is the first (required) integral of f.

The reference frame used here for the soil flow is Eulerian (Crowe and Smoot, 1979) as opposed to LaGrangian. (The Eulerian is the standard reference frame for conventional fluid flow problems.) Crowe and Smoot (1979) review the advantages of each reference frame. In Crowe's Particle-Source-In Cell model (Crowe et al., 1977), he uses the LaGrangian reference for the particles and an Eulerian reference for the airflow. The added complexity of two different reference frames is not needed here.

Model

The model boundaries consist of a general cylindrical-shaped control volume (Fig. 1), whose plan view is depicted in Fig. 2 along with the four required coordinate systems: (a) North,East; (b) x,y,z; (c) R,u,z; (d) r,z, and various angles. The coordinate systems are needed to handle the following requirements.

The North,East system is required since available wind data summaries have tabulated the horizontal wind vector component and its angle relative to North.

The description of the field surface S_1 must be given in terms of a fixed coordinate system, x,y,z. Granted, North,East could be used, but a rectangular field would not always be so oriented, hence the field angle α is required. The R,u,z coordinate system is oriented to the wind vector as shown in Fig. 2. Integration of f in this coordinate system is simpler in that the data describing the integration of f in the R direction (i.e., $\bar{q}$) is almost completely provided from wind tunnel data (i.e., q), except for an axis shift along R. The final set of coordinates, i.e., r,z, refer to the two-dimensional coordinate system utilized to develop the q functions from tunnel data.

The angles β and θ are wind vector angles referred to two different coordinate systems. β is required for calculations and θ is the angle used in the available wind data. The angles are related as

$$\beta = (\alpha - \theta) + 3\pi/2 \qquad [3]$$

The portion of equation [1] representing the mathematical portion of the model that we are interested in here is

$$\dot{m}_1 = \int_{S_1} \bar{f}_z \, dA \qquad [4]$$

which describes the soil loss rate from the eroding earth surface, S_1 (see Fig. 1). Since we do not know $\bar{f}_z$ at S_1, the soil loss flow rate $\dot{m}_1$ will be derived by the spacial integration of the continuity equation within the control volume and will result in two line integrals around the circumference of S_2, in terms of q.

The model is limited to a field whose plan view, S_1 or S_2, is restricted to a convex region, i.e., a region whose circumference would be "cut" by a straight line at no more than two points (Courant, 1936, pp. 100, 362). This restriction implies that soil, once it has left the field, will not return to the field unless the velocity vector changes direction. While it appears theoretically possible to handle a nonconvex region by the methods described later, it is not clear how this could be done considering the changing wind vector angle. Fortunately, most agricultural fields are convex, i.e., rectangles. The convex assumption is dictated by the assumption that the wind velocity vector does not vary along the top plane of the control volume.

From Fig. 1 we note that the height of the control volume is h. This represents the height wherein all soil that leaves S_2 is essentially the suspended portion and soil that leaves or enters S_3 is due to saltation and creep. It is further assumed that h is constant during the erosion process, i.e., the change in h due to soil loss is negligible compared to h. While this is not absolutely essential, it simplifies the analysis with very little loss in generality.

The model allows for the S_3 surface to have an inflow of soil in the upwind direction and an outflow in the downwind. This, in conjunction with the general convex shape, will allow the model to handle field boundaries that are both erodible and nonerodible and, obviously, nonrectangular fields.

Equation Derivation

The objective is to obtain an expression in q, since q would be available from tunnel data. The derivation proceeds from the application of the continuity equation in R,u,z coordinates with a translation of R to r. Then, since most fields will be described in the x,y coordinate frame, we transform the results to that system. Finally, to emphasize the fact that the resulting line integrals are

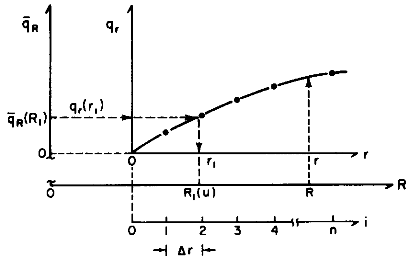

Fig. 3—A typical line-intensity function depicted with three different horizontal axes.

around a closed path, we then transform the x and y to functions of arc length s.

A typical q_r curve, which is depicted in Fig. 3, could also be determined experimentally in the field by placing Bagnold-type catchers at various r values downwind from a nonerodible boundary. The line intensity function for Fig. 3 is

$$q_r(r, h) \triangleq \int_0^h f_r(r, z)dz \quad [5]$$

(For simplicity, the J(t) independent variable will be suppressed unless needed for clarity.) Equation [5] expresses the fact that a catcher catches all the soil up to some height h at various values of r and that if the time interval of sampling is small, the amount of soil divided by the time interval and the width of the catcher approximates q_r, the integral of the horizontal component of the soil flux in the downwind direction. A similar definition describes the integral along a line on S_2 in the r direction, i.e.,

$$q_z(r, h) \triangleq \int_0^r f_z(r, h)dr \quad [6]$$

Here we are integrating the vertical component of the soil flux vector that exists on S_2, i.e., essentially the suspension component.

In the r,z coordinate system, the soil flux vector is expressed as

$$f = f_r + f_u + f_z \quad [7]$$

where

$$f_u = 0 \quad [8]$$

That is, there is no crosswind component. This is tacitly assumed by the fact that one utilizes wind tunnel experimental data. The other components are expressed as functions, i.e.,

$$f_r = f_r(r, z) \quad [9]$$

and

$$f_z = f_z(r, z) \quad [10]$$

where it should be noted that the flux does not depend on the u direction. This assumption in conjunction with the u component of flux being assumed zero, i.e., equation [8], implies that the flow in adjacent r,z planes does not interact.

The continuity equation (Bird et al., 1960, p. 75)

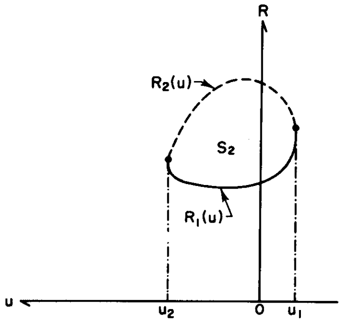

Fig. 4—A plan view of the control volume illustrating the limits of integration on S_2 in the R,u,z coordinate system, z=h.

$$\nabla \cdot \bar{f} = -\frac{\partial \rho}{\partial t} \quad [11]$$

where

$$\bar{f} = \bar{f}_R(R, u, z) + \bar{f}_z(R, u, z) \quad [12]$$

when integrated for a steady state condition in the R,u,z system becomes

$$\int_{u_1}^{u_2} \int_{R_1(u)}^{R_2(u)} \int_0^h (\nabla \cdot \bar{f})\, dz\, dR\, du = 0 \quad [13]$$

Figs. 1 and 4 illustrate the limits of integration for equation [13]. To integrate equation [13], we shall invoke Green's theorem, but first we must develop the functional relationships between r and R and q_r and $\bar{q}_R$, which will be needed later.

Fig. 3 depicts the relationship of q_r and $\bar{q}_R$. The relationship depicted implies that along the upwind edge of the control volume, i.e., $R_1(u)$ (Fig. 4), there exists an inflow $\bar{q}_R$ (R_1). This inflow, which came from a region below $R_1(u)$, is known before the integration for S_2 commences. This inflow essentially selects an r_1 and clamps the q_r and clamps the q_r curve to the R axis at R_1. In Fig. 3, this is depicted by the dashed arrow line. To find the q at any other R involves adding the difference in R to r_1 and reentering the q_r curve. This is expressed as the transformation between r and R, i.e.,

$$r = R - R_1(u) + r_1 \quad [14]$$

where

$$r_1 = \psi(\bar{q}_R(R_1)) \quad [15]$$

Equation [15] is derived by solving

$$q_r(r, z) = \bar{q}_R(R, u, z) \quad [16]$$

for the inverse of q_r, i.e., ψ. This shifting scheme is predicated on the assumption that the inflow of soil from one surface can be combined as indicated, i.e., the curve does not change because the soil entering came from a potentially different surface. If S_2 is contained within a larger eroding surface of the same type, there obviously would be no question.

Now equation [14] can be represented as

$$r = r(R, u) \quad [17]$$

and when equation [17] is substituted in equation [16], we have

$$q_r(r(R, u), z) = \bar{q}_R(R, u, z) \quad [18]$$

To integrate equation [13], we evoke Green's theorem (Kaplan, 1952, p. 242, 239) for the integration in R,z, i.e.,

$$\int_{R_1(u)}^{R_2(u)} \int_0^h (\nabla \cdot \bar{f}) dR\, dz = \oint_\gamma \bar{f}_n\, ds = \oint_\gamma \bar{f}_R\, dz - \oint_\gamma \bar{f}_z\, dR \quad [19]$$

where for our rectangular surface the line integrals become

$$\oint_\gamma \bar{f}_R\, dz = \int_0^h \bar{f}_R(R_2, u, z) dz + \int_h^0 \bar{f}_R(R_1, u, z) dz. \quad [20]$$

and

$$\oint_\gamma \bar{f}_z\, dR = \int_{R_1}^{R_2} \bar{f}_z(R, u, 0) dR + \int_{R_2}^{R_1} \bar{f}_z(R, u, h) dR. \quad [21]$$

It is important to note from the definition of line intensities (Table 2 and equations [5] and [6]) that the central term of equation [19] can be represented as

$$\oint_\gamma \bar{f}_n\, ds = \sum_{i=1}^{n} \int_{\gamma_i} \bar{f}_n\, ds = \sum_{i=1}^{n} \Delta\bar{q}_i \quad [22]$$

and, therefore, a more general concept of q is obtained. That is, q is the line integral of the normal component of the soil flux vector along *any* prescribed path. In our defining equations we imply a straight path along an axis, but because of equation [22], this is not a necessary condition. This more general definition will allow for the integration of the surface soil flux along a rough surface!

Substitution of the appropriate q_i function into equation [20] yields

$$\oint_\gamma \bar{f}_R\, dz = \bar{q}_R(R_2, u, h) - \bar{q}_R(R_1, u, h) \quad [23]$$

and into equation [21] yields

$$\oint_\gamma \bar{f}_z\, dR = \bar{q}_z(R_2, u, 0) - \bar{q}_z(R_1, u, 0) + \bar{q}_z(R_1, u, h) - \bar{q}_z(R_2, u, h) \quad [24]$$

The integration of equation [13] is completed by integrating equations [23] and [24] with respect to u and noting that these new integrals represent the mass flow rate from the various surfaces of the control volume, e.g., for S_1 we have from the $\bar{q}_z(R,u,0)$ components in equation [24]

$$\dot{m}_1 = \int_{u_1}^{u_2} \bar{q}_z(R_2, u, 0) - \bar{q}_z(R_1, u, 0)\, du \quad [25]$$

From equation [13] we note that the integral is zero; therefore, we can solve the integrated form of equations [23] and [24] for $\dot{m}_1$ and obtain equation [26], i.e.,

$$\dot{m}_1 = \int_{u_1}^{u_2} \bar{q}_R(R_2(u), u, h) - \bar{q}_R(R_1(u), u, h)\, du + \int_{u_1}^{u_2} \bar{q}_z(R_2(u), u, h) - \bar{q}_z(R_1(u), u, h)\, du \quad [26]$$

Now from the definition of a line integral (Kaplan, 1952, p. 240), we note that equation [26] becomes

$$\dot{m}_1 = -\oint_C \bar{q}_R(R(u), u, h)\, du - \oint_C \bar{q}_z(R(u), u, h)\, du \quad [27]$$

where the path C is the circumference of S_2.

By defining

$$\bar{q} \triangleq \bar{q}_R + \bar{q}_z \quad [28]$$

and substituting this in equation [27], we get

$$\dot{m}_1 = -\oint_C \bar{q}(R(u), u, h)\, du \quad [29]$$

Now it remains to convert the $\bar{q}$ into q in terms of the r,z coordinate system. This is done by noting the equivalency of the q terms as shown in equation [18]. A similar equation exists for the z "component" of q. Upon making the appropriate substitutions into equations [28] and [29], we have

$$\dot{m}_1 = -\oint_C q(r(R(u), u), h)\, du \quad [30]$$

Equation [30] implies that the tunnel-derived q functions, when integrated around the circumference of S_2 or C, yields the soil loss flow rate from S_1, the field! While equation [30] represents a usable form for determining the soil loss rate, it does not explicitly show the dependence on the wind angle, β, or the field perimeter in the nonrotating coordinates of x and y. In addition, it is advantageous, if one wants to develop a machine solution for $\dot{m}_1$, to have the independent variable as the arc length around C. Furthermore, for the use of the transformation equation, shown later as equation [31], the relationship between u and its transformed variable must be single-valued. This condition is guaranteed by relating u to the arc length, s.

All three of these conditions are accomplished by a change of the variable of integration u to s in the following manner. From Kaplan (1952, p. 199) we have for equation [30]

$$\oint_C Q(u, h) du = \oint_C Q(u(s), h) \frac{\partial u}{\partial s} ds \quad [31]$$

From Fig. 2 we see that the transformation equations from x,y,z to R,u,z are

$$u = -x \sin\beta + y \cos\beta$$

$$R = x \cos\beta + y \sin\beta \quad [32]$$

Furthermore, if one defines any point in x and y in terms of the arc length s around C, we see that

$$x = x(s),\ y = y(s) \quad [33]$$

which upon inclusion in equation [32] yields

$$u(s) = u(s, \beta) = -x(s) \sin\beta + y(s) \cos\beta \quad [34]$$

with a similar equation for R. In order to apply equation [31] to equation [30] requires an expression for the derivative of u. From equation [34] we see that

$$\frac{\partial u}{\partial s} = -\frac{dx}{ds} \sin\beta + \frac{dy}{ds} \cos\beta \quad [35]$$

The final result of applying equation [35] and [31] to [30] yields

$$\dot{m}_1 = -\oint_C q\left\{r[R(u(s,\beta)),u(s,\beta)],h,J\right\}\left\{\frac{dy}{ds}\cos\beta - \frac{dx}{ds}\sin\beta\right\} ds \quad \ldots\ldots [36]$$

where now we can see the dependence of $\dot{m}_1$ on: (a) β, the wind angle; (b) the field perimeter, C, expressed in terms of s, and (c) the field surface conditions, J. Previously, q has been referred to as a function that would be derivable from wind tunnel experimental data. We clarify this point in the following section.

Tunnel-derived q Functions

From equation [36] it is seen that the tunnel-derived line-intensity function, q, is used to determine the net soil loss rate from S_1. The significance of this can be seen by reference to Fig. 3 where a typical q_r curve is shown. An example of such a curve is used by Chepil (1957) to describe the avalanching phenomenon. While no such curve has been derived from wind tunnel data, it is theoretically possible to do so by feeding in a prescribed q_r at the inlet to the tunnel and measuring q at the outlet (Hagen, 1982). (q_z and q_R would then be computed, based on the size distribution of the sampled particles.) By varying the level of the input q_r, data would be obtained that would descibe a finite difference equation, e.g.

$$q_{r,i+1}(h) = \Delta r\, g(J,q_{r,i}) + q_{r,i}(h) \quad i = 0,1,2,\ldots,n \ldots [37]$$

where Δr is the length of the soil sample, i and i+1 represent the input and output, respectively, for the i-th distance, and g is some unknown functional relationship, depending on $q_{r,i}$ and all the factors implied by J. The solution to equation [37] is a sequence which, when plotted, would represent points on the q_r curve depicted in Fig. 3. If Δr is small compared to the expected range of R, then equation [37] can be approximated as a differential equation, i.e.,

$$\frac{\partial q_r}{\partial r} = G(J,q_r) \quad \ldots\ldots [38]$$

and the curve of Fig. 3 is its solution for a given J. A similar curve could be obtained for q_z.

APPLICATIONS

Two applications of equation [36] will be illustrated. The first involves a "scaling up" in that multiple usage of the equation is required for a nonhomogeneous field. The details have not been determined, hence only the concepts are presented.

The second application relates to the same class of fields to which the present wind erosion equation (Woodruff and Siddoway, 1965) applies, i.e., an isolated, homogeneous rectangular field, where isolated implies no soil flowing onto the field.

The case for a circular, isolated homogeneous field has also been solved, but it is not illustrated here.

Case I

Fig. 5 illustrates a hypothetical case of a rectangular field with two different surface conditions, J_1 and J_2, surrounded by an erodible region, J_3. (The J_i implies

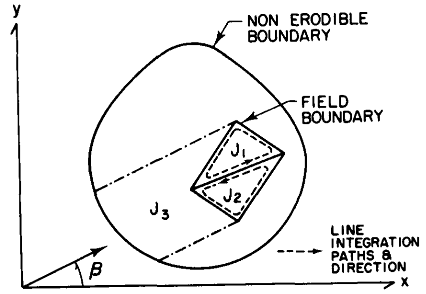

Fig. 5—A plan view of the earth surface area of Case I.

both the surface condition and the region.) Intuitatively one "sees" at least three applications of equation [36], unless the field boundary conditions implied in equation [15] can be supplied. This is not too likely, hence they must be determined by an application of only the q_r portion of equation [36] to that edge of J_3 that is upwind from the field (Fig. 5). Here it should be noted that the only application of equation [36] is for the J_1 and J_2 regions, i.e., the field for which $\dot{m}_1$ is required.

Now the sequence of calculation using multiple applications of equation [36] and q_r is dependent on β! For the case depicted in Fig. 5, we note first a solution of q_r along the common boundary of J_1,J_3 and J_2,J_3. Then, a line integral around J_2, and finally a line integral around J_1. The total $\dot{m}$ for the field is the sum of the J_1 and J_2 line integrals.

Obviously, this calculation scheme has increased in complexity over the simple case postulated by equation [36] due to the multiple application of q and $\dot{m}$, plus the determination of the region within J_3 as a function of β. The latter requirement, plus the inability to be able to describe the boundaries of J_1, J_2, and J_3 analytically, make a numerical solution mandatory.

Case II

Fig. 6 depicts a rectangular homogeneous field of size ℓ,w and surface conditions J_1 oriented at some angle α. For this case, the field and nonerodible boundaries coincide, hence there is no inflow of soil and therefore as can be seen from Fig. 3,

$$r_1 = R_1 = 0 \quad \ldots\ldots [39]$$

and the two axes coincide and are therefore equal, i.e.,

$$r = R \quad \ldots\ldots [40]$$

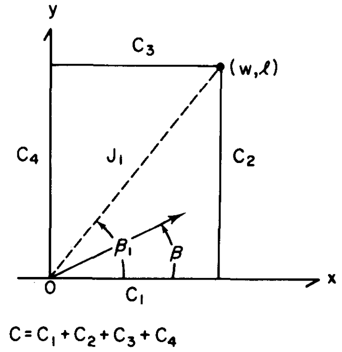

Fig. 6—A plan view of the field surface of Case II.

Equation [36] requires:

1. a description of C
2. the q (r,h,J) functions
3. r as a function of s and β
4. x(s) and y(s) and their derivatives.

Items 1, 3, and 4 can be determined from Fig. 6, although item 3 presents some difficulty. Item 2 must be supplied for a J_1 surface condition. Now if one determines items 1, 3, and 4 and substitutes these into equation [36], one arrives finally, after much tedious work, at the following equations:

$$\dot{m} = \begin{cases} 2w\,|\sin\beta|\,\langle q\rangle_{\hat{w}} - (w\,|\sin\beta| - \ell\,|\cos\beta|)\,q(\hat{w}) \quad \text{[41]} \\ \\ 2\ell\,|\cos\beta|\,\langle q\rangle_{\hat{\ell}} + (w\,|\sin\beta| - \ell\,|\cos\beta|)\,q(\hat{\ell}) \quad \text{[42]} \end{cases}$$

where

$$\hat{w} = w/|\cos\beta| \quad \text{[43]}$$

$$\hat{\ell} = \ell/|\sin\beta| \quad \text{[44]}$$

$$\ell \geqslant w \quad \text{[45]}$$

and the choice between equations [41] and [42] depends on β in the following manner. Select equation [42] if $\beta_1 \leqslant \beta \leqslant \pi\text{-}\beta_1$ or $\pi+\beta_1 \leqslant \beta \leqslant 2\pi\text{-}\beta_1$; otherwise, select equation [41] where

$$\beta_1 = \tan^{-1}(\ell/w) \quad \text{[46]}$$

These equations and the decision logic have been programmed and tested with an assumed

$$q = 1 - \exp(-0.2r) \quad \text{[47]}$$

$$w = 10.0 \text{ and } \ell = 20.0 \quad \text{[48]}$$

for a range of 38 β angles within 0 to 2π. The program compiled on a WATFIV compiler in 0.12 s and it executed in 0.07 s. The results could not be checked absolutely but appeared reasonable. They repeated themselves in a symmetrical fashion with β as expected. Values of $\dot{m}$ ranges between 10.26 and 17.59. Since no effort was made to maximize $\dot{m}$, a value larger than the observed maximum may be possible.

SUMMARY

The application of the line integral of the tunnel-derived q functions around the boundary of a field can determine the mass flow rate from the field surface. Boundary flow conditions must be either prescribed or the boundary for the problem extended to a nonerodible boundary. The solution then involves multiple application of the line integral to each homogeneous region contained within the nonerodible boundary.

The q functions required must be available from wind tunnel data that were obtained for the conditions existing on the field.

The major assumptions implied by using this method are: (a) the soil flows in parallel flow planes that do not intereact to affect the q functions, (b) the velocity vector is uniform across the field, (c) q curves can be combined sequentially down the field, and (d) the field is convex in shape.

The complete solution, i.e., the determination of soil loss, m, or E, the average flux, depends on integrating $\dot{m}$ with respect to time. That problem appears at present to be more difficult than the spacial integration covered here.

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