WIND EROSION MODELING

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INTRODUCTION

As the title implies but does not make clear, the subject of this paper is the mathematical modeling at the USDA, ARS, Wind Erosion Research Unit, Manhattan, Ks. The modeling is concerned primarily with the prediction of soil loss from agricultural fields.

We shall review briefly the wind erosion equation (model) and its modifications and from this show how our present philosophy of modeling evolved and how we expect the modeling to develop in the future. For completeness we shall also mention two other models, one that simulates the airflow around porous windbreaks, and the other, surface soil moisture.

THE WIND EROSION EQUATION AND ITS MODIFICATIONS

Although earlier versions of the wind erosion equation were published (Chepil and Woodruff 1954, 1959; Chepil 1959, 1960; Chepil et al. 1962; Chepil 1962), the base line equation, which we refer to as the wind erosion equation, is documented in Woodruff and Siddoway (1965). Some insights into the development of the equation are presented by Cole et al. (1982).

The functional form, as given by Woodruff and Siddoway (1965), is

$$E = f_1(I, K, C, L, V) \tag{1}$$

where the factors are as follows: E, the potential average annual soil loss; I, erodibility; K, surface roughness; C, climatic factor; L, equivalent field length; and V, the vegetative factor. (See Woodruff and Siddoway (1965) for a detailed description of the factors.)

A more mathematically rigorous form, as given by Cole (1982), is

$$E = f_2(V, f_3(IK, IKC, L)). \tag{2}$$

This form can be easily verified by observing how E is computed in Woodruff and Siddoway (1965).

Equation 2 represents the base line equation for the models to be discussed since they are really minor variations on how to compute either the C or L factors or how to subdivide E. In fact, the underlying theme for these changes is related to the unconscious feeling that E should be a flux and that the factors such as I, K, C, and L should vary with time. We shall return to this later when we discuss the direction of the present modeling effort; but first, let us summarize quickly the modifications made on equation 2.

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Chepil recognized the fact that all of the factors he defined could be considered to change with time and, indeed, he coped with the wind angle fluctuations (Chepil et al. 1964) by defining a prevailing wind direction angle. This angle is determined by constructing a wind erosion rose, which is a set of 16 normalized vectors whose magnitudes are proportional to the time-weighted sum of the average velocity cubed. By selecting the maximum vector that would fit within the rose, Chepil assigned the angle of this maximum vector as the angle A. He utilized this angle to compute a single value:

$$L = w \sec A \quad 0° < A < 85° \tag{3}$$

where w is the short side of the rectangular field.

Skidmore (1965) and Skidmore and Woodruff (1968) made two modifications to Chepil's method of determining L. First, they determined the prevailing wind direction by decomposing the 16 normalized vectors into tangential and normal components about an arbitrary coordinate system. The sum of the magnitudes of the normal components divided by the sum of the magnitudes of the tangential components is called the preponderance function and it was maximized by rotating this new coordinate system. The resultant angle between the x axis and east was considered the prevailing wind direction.

Skidmore (1965) did not use this angle to substitute into equation 3 to determine a single L, but instead used his prevailing wind direction angle in conjunction with his field angle and 16 vector angles to determine 16 field lengths by application of equation 3. To each length he assigned a probability, based on the relative energy computed for each L and, consequently, developed a cumulative probability density function. From this he selected a median value of L, which was designated as "the equivalent field width." This latter width was used as the L in equation 2.

A proposal to modify C to a monthly factor was put forth in Woodruff and Armbrust (1968) and Skidmore and Woodruff (1968), but it does not appear to have been used extensively.

Perhaps the most significant modification in the use of equation 2 is the partitioning of E with time, as proposed by Bondy et al. (1980). They used an erosive wind energy factor to subdivide E into periods of a fraction of a crop rotation cycle while utilizing the period values for K, L, and V. Here we note the first attempt at viewing E as a point flux rather than as an average flux and hence treating the independent variables as functions of time. It is interesting to note that Chepil et al. (1964) and Bondy et al. (1980) both used a form of an energy factor to apportion yearly soil loss. Chepil et al. (1964) apportioned within an arc and Bondy et al. (1980) within a time interval.

A computer program for the solution of the wind erosion equation (WEROS), which incorporates the preponderance concept, has been programmed in Fortran (Skidmore et al. 1970). It allows for the solution of any single variable given all of the others. With this feature, one can conceivably determine an optimum control strategy.

Another modification to the wind erosion equation was made to allow its incorporation into the EPIC (Erosion Productivity Impact Calculator) model. This consisted primarily of incorporating the erosive wind energy concept (Bondy et al. 1980) with the subroutines of WEROS (Skidmore et al. 1970), which were concerned with the solution of equation 2. The time step of 1 day required the computation of the erosive wind energy factor for 1 day. In addition, a method of handling multiple simultaneous crops was developed. For the details, see Cole et al. (1982).

OTHER WIND EROSION MODELS

Two other models have been developed. The first (Hagen et al. 1981) simulates the airflow near a porous wind barrier. This model solves five partial differential equations in order to simulate two-dimensional flow normal to a narrow windbreak. The equations are those of conservation of horizontal and vertical momentums, mass, turbulence energy, and the dissipation rate of turbulence energy. The method of solution utilizes finite-difference techniques.

The second model (Skidmore 1983, personal communication), which is still in the development stage, simulates the effect of soil moisture on the soil flow rate per unit width. This effect is represented as a cohesive stress due to moisture, which reduces the wind shear stress. The cohesive stress is determined from climatic variables--for example, air velocity, humidity, temperature, and net solar radiation--and the hydraulic properties of the soil. Preliminary validation results suggest that the model output, that is, the climatic erosivity factor, is related linearly to a calculated suspension flux.

AN ANALYSIS OF WIND EROSION EQUATION LIMITATIONS

From the previous historical review of the wind erosion equation, it should be apparent that any changes made to the base line equation, that is, equation 2, were due to the difficulty of determining single values for factors such as I, L, V, and K. This difficulty appears to have arisen because of the ambiguous methods suggested for their determination; e.g., Woodruff and Siddoway (1965) stated that "The equation actually evaluates the erodibility of a field having certain L, K, and V values in terms of what it would have been during the severe soil blowing time." In Chepil et al. (1964) we note that the prevailing wind direction used to compute L (equation 3) is based upon long-term wind distribution data, which implies more than the severe blowing season. Obviously, we have a contradiction.

If one has difficulty in selecting the factors for equation 2, then his confidence in the prediction of values of E may be quite low, especially when he visualizes that these factors would change during the yearly cycle. This feeling apparently lead to the modification previously discussed.

Reviewing these changes indicates that there was a prevalent feeling that, if the factors which affect E could be described as functions of time rather than as a single value, then the computed value of E would be more accurate. Furthermore, since the loss of soil tends to be higher during certain months of the year, a knowledge of the magnitude of the erosion process during this shorter time interval would be more useful for control practices.

This reasoning then leads to the following questions: What is E for a period shorter than a year? Is E a good measure for short periods? What is E? What is the measure of soil erosion?

To answer these questions, and many more that they engender, an analyses of the significance of E as a measure of the soil erosion process was performed. The results of this as yet unpublished study can be summarized by the following two equations:

$$E = \text{Expt.}(\langle\langle f \rangle_A \rangle_T) \qquad (4)$$

where

$$\langle\langle f \rangle_A \rangle_T \triangleq \frac{1}{AT} \int_T \int_A f \, da \, dt = \frac{m}{AT}. \qquad (5)$$

Equation 4 indicates that E is the statistical average (or mean) of the arithmetic average of the normal component of the soil loss flux vector, f (a function of time and space), where the arithmetic average is over the area A and time interval T of 1 year. It is important to distinguish between the two types of averages so that it can be made clear what is implied by the generally accepted definition for E; that is, Woodruff and Siddoway (1965) defined E as "the potential average annual soil loss in tons per acre per annum."

From equation 5 we see that the arithmetic average of f is also equal to the normalization of the mass loss, m, by the area and time interval. This average is the annual soil loss of the above E definition; that is,

$$\text{annual soil loss} = m/(A \cdot T); \quad T = 1 \text{ yr}. \qquad (6)$$

The potential average represents the statistical average, shown in equation 4.

Equation 5 emphasizes three important points. First, that the average flux is dependent on time and space intervals and not on points within the interval, whereas f is a function of points of time and space within the intervals. This leads immediately to the observation that to develop an E equation for any time interval, the flux function is generally required. Furthermore, since

$$f = f(U(t), K(x,y,t), M(t), \ldots) \qquad (7)$$

(where U is the windspeed, K some function of roughness, M a function of soil moisture, and ... implies other unspecified variables), it is at this level of definition and not at the E level that it is legitimate to apply time-varying functions to improve the accuracy of predicting E.

The third observation to be made from equation 5 is that the area and time intervals only enter into the computation of <<f>> as limits of integration and as a denominator. They are not computed. They must be specified. It is the soil flux and hence the soil lost (m) during the period T that is computed.

The importance of this is that any measure of the soil loss process depends on the soil loss mass. This further suggests that the phrase "soil loss" when used as a noun be reserved for m and when used as a process, while always sufficiently quantified by <<f>>, may be measured for special cases in other ways. For example, if one were comparing the soil loss processes for the same field and the same interval of time for two different soil conservation strategies, then a predicted m would be a necessary and sufficient measure.

TASK DEFINITION

From the above, we now see that improvement of prediction of any measure of the soil loss process is predicted upon three major tasks which are implied by equations 4 and 5: (1) the functional form of f, (2) description of the factors upon which f depends, both in time and space, and (3) methods for integration of f in time and space.

The first two are necessary for any measure of the erosion process and all three for the development of an improved wind erosion equation.

Another way to subdivide the modeling tasks is by relating the three major tasks to the concept of soil loss tolerance (τ) and a crop productivity tolerance (Π). In a sense, the soil loss tolerance is a limit placed on one part of the plant-soil system whereas a productivity tolerance is a restriction on the crop, the most significant output of the system.

If one utilizes τ as a limit, then

$$E \leq \tau \quad (8)$$

and all three of the tasks are necessary. However, if one adopts crop productivity (P) as a criterion, then

$$\Pi \leq P \quad (9)$$

where P is analogous to E in that it is an expected value of an average crop flux (kg/(ha·yr)), then only the first two tasks are required. It is worth noting that the disappearance of the third task is only apparent, since for a soil flux function to be of any use, it must tie into a comprehensive model, that is, EPIC, where the integration must be performed. The benefit of the productivity criterion is the lack of a need for a constraint on f since the constraint has been moved to a higher level, such as, crop productivity.

PRESENT MODELING EFFORTS

We have embarked on a modeling program to meet the τ criterion, that is, we consider this the immediate goal in that it is limited to the process of soil erosion and, with the exception of the integration task, all tasks also benefit the Π approach.

The development of the flux equation is envisioned at present as being quite experimentally oriented. Much of the early wind erosion research was devoted to finding q, the first integral of f with distance downwind, for various values of the surface conditions:

$$q = \int f \, dR. \quad (10)$$

The problem of time and space integration had not been satisfactorily accomplished and does not, at least conceptually, depend on experimentation except for validation. It depends more on establishing the model of a field in terms of f or q, establishing a reference coordinate system, and then determining how to perform the integration.

Spatial Integration

The task of integration has been selected as the initial research effort, and the results of the spatial integration was reported (Cole 1984).

The resulting equation, which allows calculation of the mass flow rate of soil loss ($\dot{m}$), given the appropriate q function, for any convex field is

$$\dot{m} = \oint_C q(r(R(u),u),h,J) \, du. \quad (11)$$

C is the path denoted by the perimeter of the field, u is the crosswind coordinate, h is the height of the saltation layer, r and R are downwind coordinates, and J is the set of surface properties of the field which may change with time, for example, the windspeed. Now the R,u coordinate system is relative to the wind vector, and when $\dot{m}$ is expressed in terms of a line integral around the path C (which is more amenable to machine integration), equation 11, due to a coordinate transformation, becomes

$$\dot{m} = -\oint_C q\{r[R(u(s,\beta)), u(s,\beta)], h,J\} \cdot \{\frac{dy}{ds}\cos\beta - \frac{dx}{ds}\sin\beta\} \, ds \quad (12)$$

The new variables are s, the distance along C; $\beta(t)$, a wind angle function; and x(s),y(s), the position coordinates of s relative to the x,y reference frame which is fixed to the earth. Via equation 12, one can calculate $\dot{m}$ for any convex field. Cole (1984) has developed from equation 12 the model for a homogeneous rectangular field with a nonerodible boundary. The equation for a circular field also has been developed.

As an outgrowth of this work, we are trying presently to apply a similar concept to equation 2, the wind erosion equation, to reduce the number of factors required to compute E. As part of the EPIC submodel validation, a modification of WEROS (Skidmore et al. 1976) was used to compute soil loss by periods (Bondy et al. 1980). By utilizing this modification and considering a large field to be subdividable into narrow subfields (trapezoids and triangles, or rectangles) for any wind angle, β, then the E for the field is computed as the weighted sum of the individual E's, where the weight factor is the percentage of the total area.

The advantage of this scheme is that L for the field is no longer required. Furthermore, since we now compute an E for the field for each of 16 β's, we also now compute erosive wind energy factors directly from wind distribution data, as a function of crop stage period and wind angle, rather than precomputing and entering as was done previously (Bondy et al. 1980).

For those familiar with the concept of preponderance (Skidmore and Woodruff 1968), we have eliminated the need for it by transferring the energy weighting scheme associated with preponderance into the computed erosive wind energy factors. This obviously increases the number of calculations to the point where they must be done on a computer. This model is still in the development stage.

Time Integration

In order to evaluate equations 4 and/or 5, we must make further assumptions about the flux function, that is, whether or not we consider it to be deterministic in time in the statistical sense. If considered deterministic, then we can pass on and consider its independent variables (equation 7) and pose the same questions. At present, since the functional form of f or q is in the future, we shall assume the function to be deterministic.

The question for the independent variables is not answered so clearly since some of the variables can be considered stochastic, such as the wind, and others deterministic. However, if one is dealing with a postdiction situation, even the wind can be considered determined. Consequently, when all variables are deterministic, the time integration problem is conceptually trivial and depends strictly upon an adequately sampled set of functions and sufficient storage space in computer memory. The solution to equation 4 is then equation 5 since the arithmetic average is not a random function of time.

For the prediction problem, which is what is implied when one is interested in using the wind erosion equation, it appears that unless one knows what the future functional form of all the variables will be, he will have to be satisfied with a statistical approach which treats some of the variables as random and predicts only a mean value.

The advantage of this approach is that we do not need long strings of data representing such functions as the future wind. It allows us to replace the time integration of f with the integration implied in the definition of the statistical mean for the random variables and a finite time integration for the deterministic variables; that is, from equation 4 we have

$$E = \langle \text{Expt.}(\langle f\rangle_A)\rangle_T \qquad (13)$$

where

$$\text{Expt.}(\langle f\rangle_A) = \int_{-\infty}^{+\infty}\!\!\int \langle f(U,\beta,D(t))\rangle_A \cdot p(U,\beta)\, dU\, d\beta \qquad (14)$$

and D(t) represents the set of all deterministic variables. A p(U,β) is the joint probability density function for the random functions of the wind vector, here assumed as the only random function. Other random functions also could be included as needed.

Now the time interval implied by T in equation 13 would be the period for which D(t) would repeat itself. The selection of T as this period is justified, since the time average implied by equation 13 would repeat itself with period T and with sufficient time would approach a constant value, so any further integration would be useless. The period of T in equations 4 and 5 was assumed to be 1 year; however, here its more general meaning is apparent. A typical prediction might have a T equal to the crop rotation period.

From this reasoning we see that to solve equation 13, we must replace the long-term time integration implied in equation 4 with, say, a 3- or 4-year period plus the integration of equation 14 for each time step of the interval T.

Further work is needed to determine how to handle other stochastic variables of f, such as soil moisture and its relationship to precipitation and the availability of precipitation probability density functions.

FUTURE MODELING EFFORTS

This effort can be subdivided into two parts, both of which are continuations of the tasks outlined previously.

The first is the development of the software and selection of appropriate hardware for the solution of equations 5 and/or 13. The primary tasks would be time and spatial integration and graphical input-output capability. The latter capability would allow for inputing field boundaries, nonerodible boundaries, vegetative patterns, and so forth, as well as time functions from a digital tablet. This capability would thus avoid tedious keyboard entry of certain data sets. Also, a graphical display of these time and spatial functions will be required for verification of the data entered.

Further improvements might include simplified data retrieval capability for the frequency distribution of the wind and precipitation data.

The second part of this effort, and most likely the most difficult, will be the determination of the functional form of f, that is, equation 7, or more realistically, q, its downwind integral. It is not clear at this point how this functional form will be determined; however, a polynominal fit of some type may be required similar to the Group Method o Data Handling of Ivakhnenko (Tamura and Halfon 1980)

A further logical extension of equation 7 would be the determination of the distribution of f by aggregate size. This extension comes about as a result of visualizing the wind erosion process as affecting the soil-plant system by both selective and total soil loss (Lyles et al. 1983).

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