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SYSTEMATIC PREDICTION OF WIND EROSION: PHASE 1

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INTRODUCTION

Systematic prediction of soil loss due to wind erosion has been shown to involve the time and space integration of the normal component of the surface soil loss vector (Cole 1984). The general spacial integration techniques that were developed in that paper are applied here for a circular field. This field shape was chosen to demonstrate the technique because the mathematics and resulting equations are relatively simple. Application to complex field shapes requires numerical integration and data entry techniques which are unique to digital computers. The development of this machine solution capability will represent phase 2.

METHOD

The general mass-flow-rate equation developed for any convex-shaped field is given (Cole 1984) as

$$\dot{m} = - \oint_C q\{r[R(u(s,\beta)), u(s,\beta)], h,J\} \cdot \left\{\frac{dy}{ds}\cos\beta - \frac{dx}{ds}\sin\beta\right\} ds \tag{1}$$

(see the list at the end of the summary section and figures 1 and 2 for symbol definitions).

In order to use this equation, we must define the boundary of the field in terms of s, the distance along the perimeter. Furthermore, the soil loss line intensity function q (see fig. 1), which is assumed available from wind tunnel studies, is a function of r, the distance from a nonerodible boundary. From equation 1 and figure 2 it can be seen that ultimately q, via a series of axis transformations, becomes a function of s, and β the wind angle. Figure 2 illustrates the relationship between the wind oriented coordinates R,u and the fixed coordinates x,y by which the field shape, C, is described.

To simplify equation 1 requires a mathematical description of the compound function of r, that is,

$$r[R(u(s,\beta)), u(s,\beta)] \tag{2}$$

and

$$\frac{dy}{ds}\cos\beta - \frac{dx}{ds}\sin\beta \tag{3}$$

where x(s) and y(s) are a parametric description of the field boundary and the h,J have been suppressed for simplicity in equation 2 since they do not affect the derivation. For this particular field shape, it is expedient to convert the s distance to ω via the following definition of an arc:

$$\omega = s/a. \tag{4}$$

We shall first develop equation 2 by starting with the following equation (which is derivable from fig. 1) relating r to R:

$$r = R - R_1(u) + \Psi(R_1(u),u). \tag{5}$$

Equation 5 represents the shifting of the r axis along the R axis due to two causes. The R_1 term is the shift due to the field boundary at the inflow side and Ψ the shift due to the magnitude of the soil inflow. Both of the effects are independent; that is, even if the inflow were zero (that is, $\Psi = 0$), the value of q(r) would vary due to the position of the field boundary $R_1(u)$. A more detailed explanation of equation 5 is developed in Cole (1984).

We are interested in constraining R to the boundary of our field, since this is where the inflow and outflow exist. We note from figure 2 that R as a function of u is multivalued and, as such, it is not useful for integration until it is made single valued. We do this by subdividing the perimeter of the circle into two functions depending on ω:

$$R = \begin{cases} R_2(u) & 0<\omega\leq\pi \\ R_1(u) & \pi<\omega\leq 2\pi. \end{cases} \tag{6}$$

Substitution of equation 6 into 5 yields

$$r = \begin{cases} R_2(u) - R_1(u) + \Psi(R_1(u),u) & 0<\omega\leq\pi \\ R_1(u) - R_1(u) + \Psi(R_1(u),u) & \pi<\omega\leq 2\pi \end{cases} \tag{7}$$

We see that to evaluate equation 7 requires the description of $(R_2 - R_1)$ and R_1, since Ψ, the inverse of q(r), is known.

From figure 2 it can be seen that $(R_2 - R_1)$ is any chord that intersects the circle and is parallel to the R axis. From trigonometry we have

$$R_2 - R_1 = 2a \sin \omega(s). \tag{8}$$

In order to determine R_1, the second unknown in equation 7, we will utilize one of the coordinate transformation equations between the x,y and R,u coordinate systems,

$$R = x\cos\beta + y\sin\beta. \tag{9}$$

To determine R_1 we must constrain equation 9 to the perimeter of the circle by causing the x,y coordinates to be the set of coordinates describing the circle in terms of s, that is,

$$R = x(s)\cos\beta + y(s)\sin\beta. \tag{10}$$

The analytic expression for x(s) and y(s) can be determined from figure 2 by application of trigonometry as

$$x(s) = j + a\sin(\omega + \beta),$$
$$y(s) = k - a\cos(\omega + \beta). \tag{11}$$

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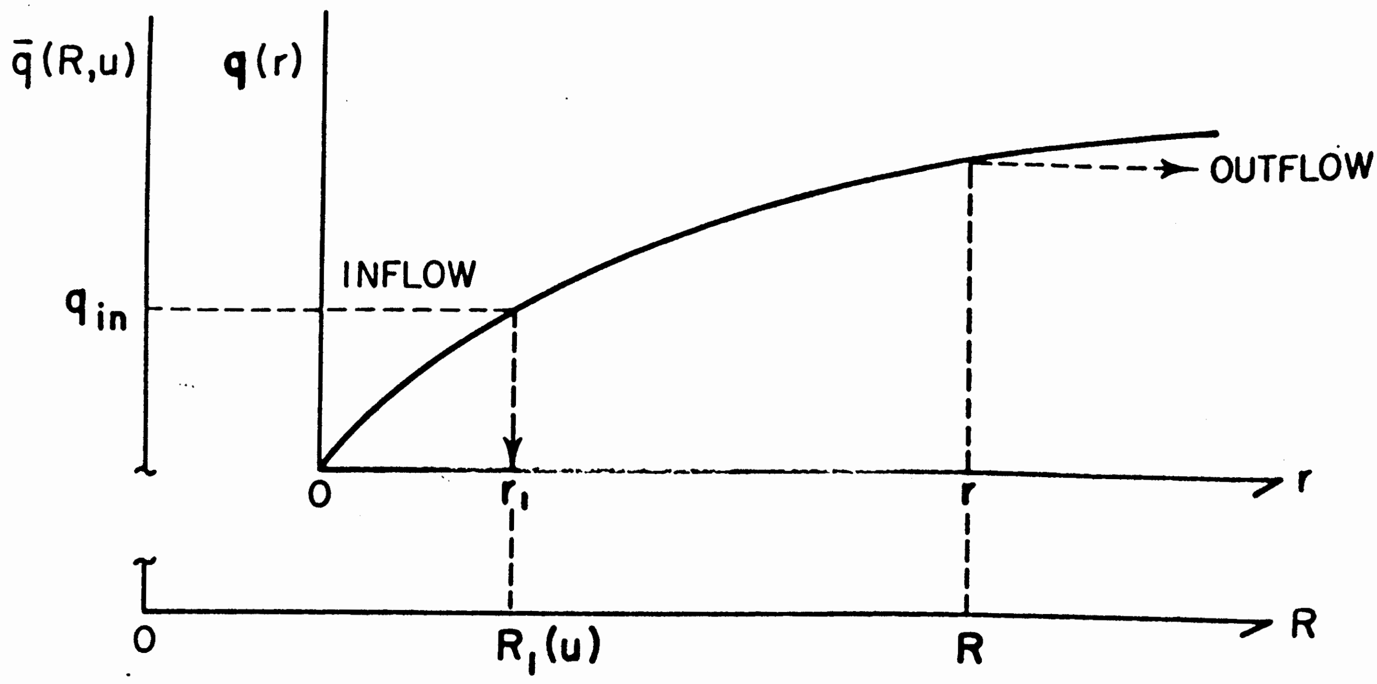

Figure 1.--Soil loss line intensity function q, relative to two axes, r and R.

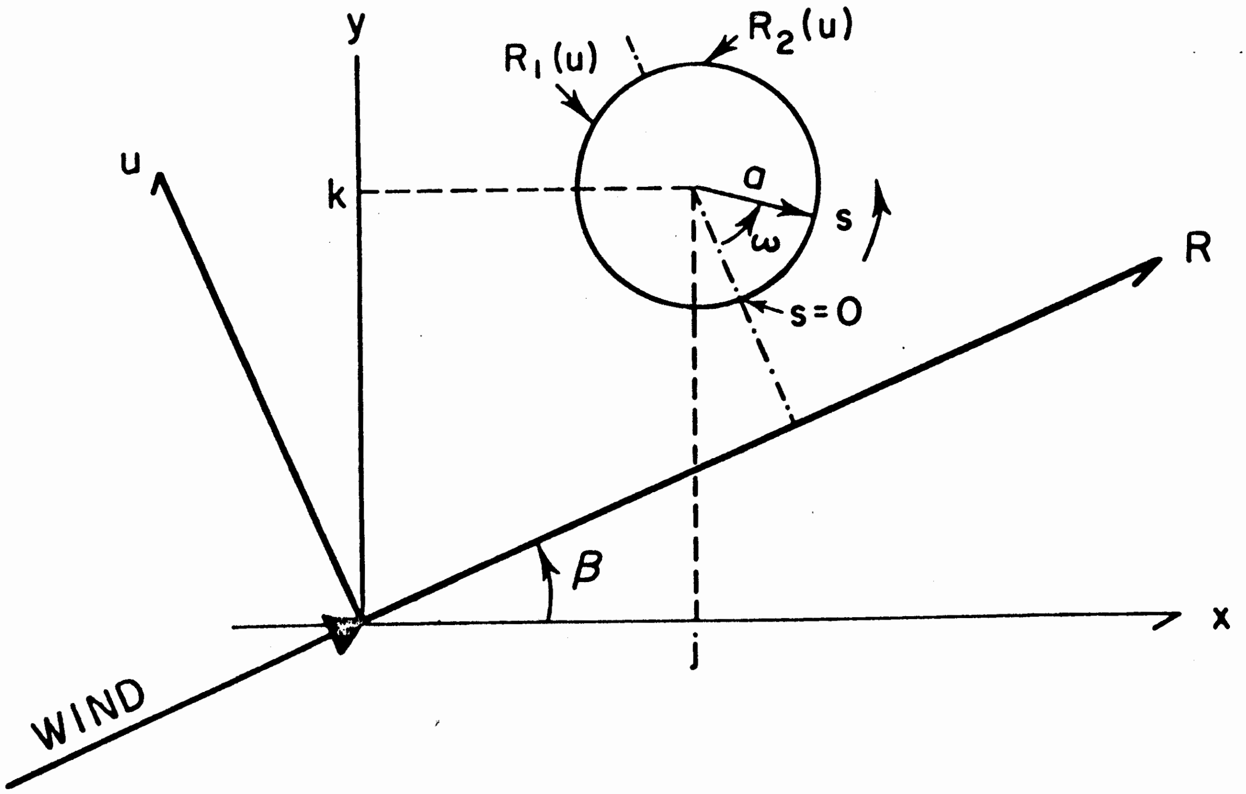

Figure 2.--Field shape, functions, and coordinate systems.

Substitution of equation 11 into 10 yields

$$R = j \cos \beta + k \sin \beta + a \sin \omega(s). \qquad (12)$$

Now equation 12 does not yet describe R_1. This is accomplished by forcing R, which describes the total perimeter, to describe only the R_1 portion, that is,

$$R_1 = j \cos \beta + k \sin \beta - a|\sin \omega(s)|. \qquad (13)$$

Now substitution of equations 8 and 13 into 7 yields the following equivalent of equation 2:

$$r = \delta\, 2a \sin \omega + \Psi(j \cos \beta + k \sin \beta - a|\sin \omega|) \qquad (14)$$

$$\text{where } \delta = \begin{cases} 1 & 0<\omega\leq\pi, \text{ outflow} \\ 0 & \pi<\omega\leq 2\pi, \text{ inflow.} \end{cases}$$

To complete the evaluation of the components of equation 1, we must evaluate equation 3. This is done by first evaluating the derivatives of x(s) and y(s) (utilizing equation 11) and substituting the derivatives into equation 3. The results are

$$\frac{dy}{ds} \cos \beta - \frac{dx}{ds} \sin \beta = \sin \omega(s). \qquad (15)$$

Substituting equations 14 and 15 into equation 1 and using equation 4 to determine $d\omega$ in terms of s results in the following mass-flow-rate equation for a circular field:

$$\dot{m} = \int_0^{2\pi} q(\delta\, 2a \sin \omega + \Psi(j \cos \beta + k \sin \beta - a|\sin \omega|), h, J)\, a \sin \omega\, d\omega \qquad (16)$$

where δ is defined in equation 14.

Now equation 16 can be further simplified if we assume a zero soil inflow condition (that is, assume that the boundary of the field is nonerodible), then

$$\dot{m} = \int_0^{\pi} q(2a \sin \omega, h, J)\, a \sin \omega\, d\omega. \qquad (17)$$

EXAMPLE

To demonstrate the utility of equation 17, we shall calculate $\dot{m}$ for a typical center pivot irrigation system using published q curve data (Chepil 1957, Fig. 1, curve d). To simplify the integration, we represent this curve as

$$q = \begin{cases} \alpha r & r < r_0 \\ \alpha r_0 & r \geq r_0 \end{cases} \qquad (18)$$

where r_0 is the breakpoint of this piecewise linear representation and α is the slope which is assumed constant.

Substitution of equation 18 into 17 results in two integrals. These two integrals result in this case because the numerical values r_0 and a are such that

$$0 < r_0 < 2a. \qquad (19)$$

This results in two regions for ω, which are separated by ω_0, where

$$\omega_0 = \sin^{-1}(r_0/2a). \qquad (20)$$

Since the soil loss rate is equal for each half of the circle, we integrate over one-half the circle and double the results, that is,

$$\dot{m} = 2\left(\int_0^{\omega_0} \alpha r\, a \sin \omega\, d\omega + \int_{\omega_0}^{\pi/2} \alpha\, r_0\, a \sin \omega\, d\omega\right) \qquad (21)$$

where

$$r = 2a \sin \omega \qquad 0<\omega\leq\omega_0. \qquad (22)$$

Integration of equation 21 results in

$$\dot{m} = 2\alpha(a^2\omega_0 + \cos \omega_0(ar_0 - a^2 \sin \omega_0)). \qquad (23)$$

The numerical values from Chepil's curve (Chepil 1957) expressed in SI units are

$$\alpha = 7.158 \times 10^{-4}\ t/(m^2 \cdot h)$$

and

$$r_0 = 502.9\ m.$$

For a typical center pivot irrigation field on a 1/4 section,

$$a = 402.5\ m.$$

From equation 20 we find that

$$\omega_0 = 0.675 \text{ radians.}$$

Finally, substitution of these four values into equation 23 yields

$$\dot{m} = 270\ t/h$$

as the rate of soil erosion.

SUMMARY

A method for incorporating a specific field shape into the general mass-flow-rate equation has been demonstrated. The resulting equation (equation 16) allows for the use of a single line intensity function, which has been shifted and transformed appropriately. The equation considers not only the surface conditions implied by J but also the wind angle, radius of the circle, the offset distances j and k, and the magnitude of the soil inflow.

We note that if there is no inflow--that is, if the field boundary is nonerodible--then the mass flow rate is independnet of the wind angle.

This method, while practical for simple geometric shapes, becomes quite impractical for nonanalytical shapes, and the numerical integration of equation 1 must be performed.

Symbol	Definition and dimensions[2]
a	radius of circle, L
C	the perimeter of the field surface, L
h	distance from soil surface to top of the field control volume. This also may be considered the saltation height, L.
j	x coordinate of the center of the circle, L
J	the set of surface conditions which affect q
k	y coordinate of the center of the circle, L
$\dot{m}$	the soil mass-flow rate through a specified surface, $M\ T^{-1}$
q	line intensity, the soil flow rate per unit width, $M\ L^{-1}T^{-1}$
$\bar{q}$	same as q but with respect to the R,z axis, $M\ L^{-1}T^{-1}$
r	distance along the r axis, L
r_0	breakpoint of q in equation 18, L
R	distance along the R axis, L
s	arc length of perimeter C, L
u	distance along the u axis, L
x	distance along the x axis, L
y	distance along the y axis, L
α	slope of linear part of q in equation 18, $ML^{-2}T^{-1}$
β	wind angle, the angle of the wind relative to the positive x axis, counterclockwise positive (see fig. 2), dimensionless
δ	defined in equation 14, dimensionless
π	3.14159..., dimensionless
Ψ	the inverse function of q(r)
ω	see equation 4, dimensionless
ω_0	see equation 20, dimensionless

Subscripts

i	index, 1, 2, 3 ... various surfaces and/or arc lengths

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[2] M, L, and T as dimensions refer to mass, length, and time.

REFERENCES

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