A Stochastic Formulation of Soil Erosion Caused by Wind

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ABSTRACT

DEVELOPMENT of a new equation to predict soil erosion requires the integration of the surface soil flux vector across the eroding area and for some interval of time. In this paper we analyze the time integration and show that both an arithmetic and statistical average must be considered for prediction.

The arithmetic average, which is called the soil erosion, is shown to be a measure which may or may not be random. When random, its statistical mean is called the average soil erosion. For this average, consideration must be given to two time intervals, a soil loss accounting interval and a periodicity associated with the deterministic independent variables. This latter average is shown to be identical with the present measure of wind erosion when the two time intervals are 1 year.

A general equation for the statistical average is developed and its use is illustrated by developing specific equations for two different soil loss accounting intervals.

INTRODUCTION

Prediction of wind erosion soil loss from agricultural fields has been possible since 1965 by using the wind erosion equation (Woodruff and Siddoway, 1965). This equation resulted from the research of W. S. Chepil. The wind erosion equation accounts for the variability of predicted soil loss between fields because of differences in location, soil type, crop, surface roughness, and to a certain extent the size of the field. The major problem when using this equation is the required selection of a single value for each factor when it is observed that the factors change in time. The second major problem is that the predicted variable, E_c,* which is referred to as "The potential average annual soil loss in tons per acre per annum . . ." (Woodruff and Siddoway, 1965) is viewed as changing in time due perhaps to a crop rotation cycle of several years. Several authors have suggested methods to overcome these apparent deficiencies by using the equation in conjunction with erosive wind energy weight factors (Bondy et al., 1980; Skidmore and Woodruff, 1968).

Cole (1984) has reported the initial stage of research, which is directed toward the development of an improved method of predicting soil erosion due to wind. The method is based on the principle of conservation of mass. In that paper the fluid mechanics concept of a mass flux vector was applied to predicting soil erosion by wind, and it was shown how spatial integration of the flux function could be conceptually accomplished to allow calculating $\dot{m}$, the soil loss flow rate.

In this paper we complete the integration process by integration $\dot{m}$ with respect to time. This temporal integration, which allows computation of the mass of the soil loss, m, eventually leads to the computation of statistical and arithmetic averages. These averages are implied by the definition of E_c, the accepted measure for the soil erosion process caused by the wind. In the development of this prediction method, we shall find it necessary to differentiate between these averages and to allow time intervals of soil loss other than a year. This variable time interval measure has been noted as desirable for estimating the magnitude of soil erosion during the interval of time when high erosion losses are expected (Bondy et al., 1980).

SOIL EROSION MEASURES

To develop an improved method or equation for prediction, one must know what is to be predicted. For the case of soil erosion prediction as it relates to the total soil loss from a field, the obvious place to start is with the present wind erosion equation (Woodruff and Siddoway, 1965). The definition cited earlier, i.e., potential average annual soil loss, alludes to a time average of soil loss. The word potential implies the idea of uncertainty or randomness of the erosion process. Cole (1984) has hypothesized from this definition that

$$E_c = \frac{1}{AT} \int_T \int_A f_z \, dA \, dt \quad \text{. . . . . [1]}$$

where f_z is the normal component of the surface soil flux vector and T and A are time and space intervals during which the mass of soil, m, is lost. This flux vector is a time and space differentiation of m (Courant, 1936), i.e.,

$$f = f(x,y,z,t) = \lim_{A,T\to 0} \left(\frac{m}{AT}\right) \quad \text{. . . . . [2]}$$

It is a conceptual device which allows calculating the soil loss using the methods of calculus and fluid mechanics. By integrating f_z as shown in equation [1], we determine the total mass transported from the field.

From equation [1] it can be seen that E_c represents a time *and* space average of f_z and that the dimensions of f_z and E_c are identical, i.e., mass/(area·time). Consequently, one cannot differentiate between f_z and its average by their dimensions or units. The major difference between them is that E_c is a function of the

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*E, the dependent variable of the wind erosion equation, is referred to in this paper as E_c to avoid confusion with the expectation operator of statistics, i.e., E(·).

intervals of area and time whereas f_z is a function of the *points* within these specified intervals and that for given A and T intervals there is only one value of E_c whereas f_z has many values.

Now this may seem quite academic; however, it is essential to understand that f_z can change with time but E_c cannot, once the interval of averaging is selected.

Recall that one problem with the wind erosion equation was the selection of single values for each factor as they changed in time! The solution to this problem becomes obvious when we consider that similar factors are the dependent variables of f_z, i.e.,

$$f_z = f_z(\hat{J}(x,y,t), x,y,0) \ldots\ldots [3]$$

where

$$\hat{J}(x,y,t) \triangleq \{\hat{I}(x,y,t), \hat{K}(x,y,t), \hat{V}(x,y,t), \hat{U}(t), \hat{M}(x,y,t), \ldots\} \ldots\ldots [4]$$

Since f_z is a function at least of the soil type (I), soil roughness (K), vegetation (V), wind velocity (U), and precipitation (M), which do vary in time and space, equation [1] can accommodate this variability both in time and space. Of course the calculation of E_c becomes more complex because one must now describe these factors as functions rather than constants; however, there appears to be no alternative.

STATISTICAL AND ARITHMETIC AVERAGES

Initially (Cole, 1984) it was thought that equation [1] was an accurate expression for E_c; however, further analyses has indicated that equation [1] does not consider the uncertainty implied in the definition of E_c; i.e., *potential* average annual . . .

As it stands, the right-hand side of equation [1] is an accurate representation of an *arithmetic* average of a deterministic function. By deterministic we imply a function in the sense of the differential calculus as opposed to the random variable of statistics. The variables $\hat{J}$, A, and T and the function f_z must be specified in advance to be considered deterministic. This, however, is not the case when *predicting* erosion, at least with regard to the wind velocity and precipitation components of $\hat{J}$, which are random.

To compute soil erosion when all the variables are known or predict the average soil erosion when some of the variables are random, we must differentiate between the arithmetic and the statistical average. We use w to represent the double arithmetic average, i.e.,

$$w \triangleq \frac{1}{AT} \int_T \int_A f_z \, dA \, dt \ldots\ldots [5]$$

and W to represent the statistical average of w, i.e.,

$$W = E(w) \ldots\ldots [6]$$

To fully appreciate the difference between w and its expected value, we must understand why w can have the following two interpretations.

For the deterministic case when all input variables are determined, then w is also deterministic and the calculation of soil erosion is determined by equation [5]. This would be the case, for example, when one has performed an experiment to validate the concept of the time and space integration of f_z as a means of calculating w for various shaped fields. Generally, the time interval will be limited by how much data can be stored to adequately represent the time variations of the independent variables.

The second usage of w implies that one or more independent variables of f_z are random, e.g., wind velocity. Hence even though f_z is considered a deterministic function, the values of w that can be computed with some random inputs are also random. If one knows the joint probability density function of the random input variables, he can, via a combination of equations [5] and [6], compute the mean of the probability density function of w, i.e., W. We shall develop the general equation for W later.

In summary, we now see that w can represent a time and space average in an arithmetic sense or, alternatively, as a single sample from a population whose statistical mean is W. This latter meaning is implied by E_c, Chepil's measure of soil erosion, when T, the soil loss interval, is 1 year. As will be seen later, W is a more general wind erosion measure than E_c, but before this can be seen, we must investigate some of the general concepts of stochastic processes as they relate to wind erosion soil loss, delineate the various time intervals implied by equation [5], and utilize the results of the spatial integration of f_z, i.e., $\dot{m}$, the soil loss flow rate (Cole, 1984).

SPATIAL INTEGRATION

The spatial integration of f_z can be symbolized as

$$\dot{m} = \dot{m}\ (\beta(t), J(t), C) = \int_A f_z \, dA \ldots\ldots [7]$$

where the soil flow rate is shown as functionally dependent on $\beta(t)$, the wind velocity angle; C, the perimeter of the region of integration; and J(t), the spatial homogeneous surface characteristics, i.e.,

$$\hat{J}\ (x,y,t) = J(t) = \{ I(t), K(t), V(t), U(t), M(t) \ldots \} \ldots [8]$$

(Equation [7] is a simplification of the detailed notation used in Cole (1984) and is adequate for our present objective.)

The assumption of a homogeneous region does not limit our ability to deal with spatial inhomogeneity, since equation [7] implies, but does not show, that it is the net loss from a region which can have an inflow from an adjacent region. Hence by conceptually summing the $\dot{m}$ from each homogeneous subregion, we can compute an $\dot{m}$ for an inhomogeneous field.

The following three points are necessary for simplifying the development of W:

1. Rather than deal with the added complexity of a subscripted $\dot{m}$, we shall assume that this can be done and hence deal with the $\dot{m}$ as a simple subsection of a field.

2. Since the spatial and temporal integration of f_z results in m, it is easier to work with m until we develop the equation for the average soil loss. Then we make the transition to soil erosion, w, via equation [10]. Equation [10] follows immediately from a modification of equations [5] and [6] into equations [9] and [10], respectively, i.e.,

$$w = \frac{m}{AT} \ldots\ldots [9]$$

where any m is considered random. Hence from equation [6] when applied to [9] we have

$$W = E(w) = \frac{1}{AT} E(m) \ldots\ldots\ldots\ldots\ldots\ldots\ldots [10]$$

3. The independent variables in equations [7] and [8], not including C, are subdivided into two classes, i.e., a deterministic set

$$D(t) = \left\{ I(t),\ K(t),\ V(t), \ldots ? \right\} \ldots\ldots\ldots\ldots [11]$$

and a random or stochastic set

$$S(t) = \left\{ U(t),\ \beta(t),\ M(t), \ldots ? \right\} \ldots\ldots\ldots\ldots [12]$$

One could argue for a different combination; however, the principles would still be the same. The choice here is reasonable, i.e., those in S represent things of which we have absolutely no knowledge except in a statistical sense whereas those in D represent some degree of control or knowledge where a reasonable guess can be made as to their future functional form. This regrouping of variables allows us to represent the total mass from the region as

$$m = \int_T \dot{m}\ (D(t), S(t))\ dt, \ldots\ldots\ldots\ldots\ldots\ldots [13]$$

where we temporarily suppress C for clarity.

THE STOCHASTIC EROSION PROCESS

The concept of periodicity and the time intervals associated with the erosion process are essential when considering soil erosion as a stochastic process. We clarify these ideas in the following two sections and then develop an expression for E(m) which, with equation [10], yields W.

Periodicity

To deal with m in a predictive sense, we are forced to accept the assumption of stastical regularity which is that "There are many repeating situations in nature for which we can predict in advance from previous experience roughly what will happen, or what will happen on the average, but not exactly what will happen" (Davenport and Root, 1958). The importance of this concept is that for whatever the interval of time that "the situation" (or trial) exists, before it repeats itself again, all conditions that are determinable must be the same between trials. In our case, since we are losing soil from a given field over many years, the trials are sequential in time, which implies that all deterministic variables must be periodic. We signify this as

$$D(t) = D(t + \tau) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [14]$$

where τ is the time duration of the trial. For our case,

$$\tau \geqslant 1 \text{ year} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [15]$$

and must be a multiple of a year. This restriction on τ is dictated by the implied basic periods in D, due to plant vegetative cover, tillage practices, etc.

One might argue that we cannot predict what crops will be growing 30, 40 or 50 years into the future; however, one must eventually come to grips with the fact that the use here of the statistial mean implies periodicity and hence some period must be chosen, perhaps even 30 years or more. Generally, τ will be equal to the crop rotation period which may vary from one to several years. However, it must be remembered that τ is dictated by the combination of variables that make up D, and not just a single component such as the crop sequence.

The other manifestation of periodicity occurs in S and is due to the period of available weather data, i.e., wind velocity and precipitation. This is generally the year, although the smallest time interval for which wind frequency data are available is the month. From this it can be surmised that the probability density function of the horizontal component of the wind vector is not stationary within the year but is assumed stationary within the month. Because of the pooling of multiple year wind data, the assumption is that the probability distribution is periodic with a period of 1 year and stationary within the month.

We symbolize the periodicity by

$$p(S,t) = p(S,t + 1 \text{ yr}) \ldots\ldots\ldots\ldots\ldots\ldots [16]$$

and the stationarity as

$$p(S,t) = \left\{ \begin{array}{l} p_1\ (S),\ \text{January} \\ p_2\ (S),\ \text{February} \\ \quad '' \qquad '' \\ \quad '' \qquad '' \\ \\ p_{12}\ (S),\ \text{December} \end{array} \right\} \ldots\ldots\ldots\ldots [17]$$

Here then we see the possibility of two periods inherent in the input variables, 1 year and multiple years.

One further assumption which is implied by equation [14] is that any deterministic functions of the process, e.g., soil erodibility or crop biomass, are the same for every τ in the future. Obviously, if the erosion process were to go on indefinitely without some form of soil renewal, these functions would change between trials. While this may seem quite restrictive, the only other alternative involves allowing τ to approach infinity and then one either specifies D(t) for all time or uses a simulation model which computes D(t). The Erosion Productivity Impact Calculator (Williams et al., 1982) is such a model. However, to arrive at a statistical average would still require many multiyear simulations using a stochastically generated S(t).

The periodicity assumption is no worse than the tacit assumption required when selecting a single value for each variable in the present wind erosion equation, and it appears at present to be necessary.

Soil Loss Accounting Interval

Another important period or interval is that associated with the accounting of the loss of soil. This is the interval for which one wishes to know the average soil loss. Its selection does not imply that soil is not lost during any other time intervals, only that one is interested in the soil loss during this particular interval. Furthermore, since T is selectable, it may be of any duration, e.g., a month, several years, etc.

This time interval is the same as the interval of integration shown in equation [13], i.e., T. We refer to T

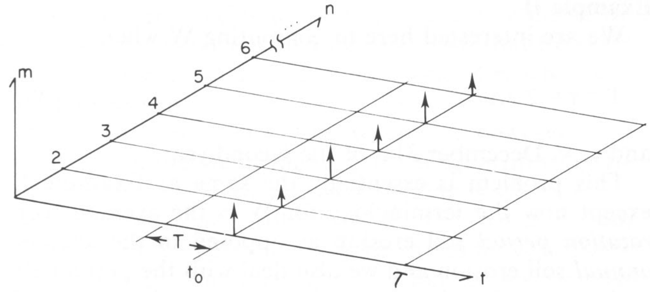

Fig. 1—The mass at time t_0, accumulated during the interval T for n trials of duration τ.

as the soil loss accounting interval and differentiate it from τ as follows:

$$0 < T \leqslant \tau \quad [18]$$

For the present measure of soil erosion, i.e., the E_c of the wind erosion equation (Woodruff and Siddoway, 1965),

$$T = \tau \quad [19]$$

The need to differentiate T from τ arises when one wishes to predict the mean soil loss for a subinterval of τ, e.g., the mean soil loss for the month of March. Furthermore, the average soil loss for a longer interval most likely must be computed from the monthly soil loss averages, since the available wind distribution data are only considered stationary for a monthly interval.

Fig. 1 is useful for differentiating these time intervals. It depicts the mass of soil, m, accumulated during T for n trials as a series of vertical arrows. The time, t_0, implies the time during the interval, τ, when the integration or soil accounting ends. Consequently, equation [13] is depicted more precisely as

$$m = \int_{t_0 - T}^{t_0} \dot{m}\,(D(t+\tau), S(t))\, dt \quad [20]$$

The combination of t_0 and T are required to uniquely specify say the month of March. No conceptual difficulties are presented if τ were 3 years and the accumulation interval were specified as all months of March; then one would have three integrals like equation [20].

Average Soil Loss

Having introduced the various times and how they relate to m (equation [20]), we now show how the definition of the statistical mean and joint probability density function allows prediction of the average soil loss and, finally, the average soil erosion, W.

To compute the statistical time mean of m when m is considered a continuous random variable, one may, for the case shown in Fig. 1, use the definition of the statistical mean (or expected value), i.e.,

$$E(m) = \int_{-\infty}^{+\infty} m\, g(m,t_0)\, dm \quad [21]$$

where g is the probability density function (pdf) of m. As shown by Davenport and Root (pages 46, 48, 1958), the more general form of equation [21] when the functional form of m is a deterministic function of random variables may be depicted (with a slight adaptation for our case) as

$$E(m(t_0)) = \int_{-\infty}^{+\infty} m(S,t_0)\, p(S,t)\, dS \quad [22]$$

Recalling that S is a set of random functions, we note that the number of integrations implied in equation [22] is greater than one. For example, if S consists of wind speed, wind direction, and precipitation, then we have a triple integral and p represents the joint pdf which, as shown in equation [22], could change with time.

The use of g in equation [21] and p in equation [22] implies that, as the number of trials, n, grows without bound, the estimate of the pdf derived from the trials (e.g., wind distribution data) would approach the pdf of the population. Since m is finite (soil can't be lost forever), one might argue that as n aproaches infinity, the soil loss for an interval will become zero and, as a consequence, the expected value of m, E(m), will also approach zero! We shall assume that E(m) will approach a finite limit, other than zero, long before the supply of soil is exhausted, and hence we will ignore all trials after soil exhaustion.

In order to complete the determination of the equation for E(m), it is desirable from the computational point of view to have the integrations implied by equation [22] nested within the time integrations that are implied by the definition of m in equation [20]. This is readily accomplished by noting that the operations of integration in time commute with the integration implied in the expectation of m, i.e., equation [22] (Davenport and Root, page 65, 1958).

We accomplish this nesting of integrals by substitution of equation [20] into [22], yielding

$$E(m) = \int_{t_0 - T}^{t_0} \int_{-\infty}^{\infty} \dot{m}\,(D(t+\tau), S)\, p(S, t + 1\text{ yr})\, dS\, dt \quad [23]$$

subject to the constraints,

$$0 < t_0 \leqslant \tau \quad [24]$$

$$0 < T \leqslant \tau \quad [25]$$

$$T \leqslant t_0 \quad [26]$$

These constraints are derivable from Fig. 1.

Equation [23] shows how the average soil loss can be calculated by integrating the mass flow rate-pdf product over the specified intervals. The first integration is over the soil loss interval T, perhaps a month, or a crop rotation period of a few years.

Equation [25] indicates that T can approach τ. The second integration is over the range of the joint pdf for the stochastic variables considered, e.g., the wind vector and precipitation. A third integration, which is not shown explicitly, is implied by the definition of $\dot{m}$ in equation [7]. Cole (1984) has shown that $\dot{m}$ can also be represented as a closed path line integral around the perimeter of the field or subregion of interest.

We have now developed the equation which is essential for predicting the average soil loss for any time interval, T, within τ. Application of equation [10] in conjunction with equation [23] yields the equation to predict W, the average soil erosion. By allowing $T \rightarrow \tau$, we can predict the average soil erosion for a crop rotation period and, if both T and τ were 1 year, then an average yearly soil erosion.

A simpler functional notation for the average soil erosion which emphasizes the various time and space intervals and other parameters which affect the predicted value is

$$W = W(A, C, t_0, T, \tau, P) \quad [27]$$

where

$$P \triangleq \{\text{parameters of S and D}\} \quad [28]$$

With this notation, it is evident which average is being considered via t_0, T, and τ. Also, we note the dependence on field shape and size by A and C and the surface conditions by P.

APPLICATIONS

Although equation [27] (or equivalently equations [10] and [23]) represents the most general concept of a soil erosion average, it is not useful for computations until one puts bounds upon it by specifying which particular average is desired, e.g., the average March soil erosion for a crop rotation cycle of 4 years or perhaps the average crop rotation soil erosion for a τ of 2 years, . . . etc. Different but similar equations will result for each case as each case is translated into unique time and time intervals, i.e., t_0, T, and τ.

To show the utility of equation [27], we offer three examples of how it can be used. (Of course the obvious and most important use is the prediction of a numerical value for an average soil erosion, but without the equations for the line intensity function (Cole, 1984) which are needed to determine $\dot{m}$, this cannot be demonstrated here.)

The first example shows the relationship between E_c of the present wind erosion equation and W. The second example develops the equation for the average crop rotation period soil erosion when monthly pdf's are available. This form is expected to be the most useful. Finally, we develop the equation for an average monthly soil erosion.

Example I

The wind erosion equation's dependent variable is referred to in Woodruff and Siddoway (1965) as "The amount of erosion, E_c, expressed in tons per acre per annum . . ." and later in the same paper as ". . . E_c, the potential average annual soil loss in tons per acre per annum . . ." From this it is reasonable to deduce that E_c can be equated to W (in the form depicted in equation [27] as

$$E_c = W(A, \text{Rectangle}, 1, 1, 1, P) \quad [29]$$

We see then to compute E_c using equations [10] and [23] that T, the soil loss accounting interval, and τ, the crop rotation period, become 1 year, and t_0, the end point of the interval T, is December 31st. Also, the perimeter of the field, C, is that of a rectangle, and A is the computed area. From this we can conclude that E_c is a special case of W!

Example II

We are interested here in computing W when

$$T = \tau = 2 \text{ yrs} \quad [30]$$

and t_0 = December 31st of the second year.

This problem is essentially the same as Example I, except now the terminology for W is the average *crop rotation period* soil erosion as opposed to the average *annual* soil erosion and we also deal with the periodicity and stationarity of p(s,t), as shown in equations [16] and [17], since $\tau > 1$ yr.

As depicted in Fig. 1, T represents a single continuous interval. This is not essential for the definition of T given earlier. It can represent the sum of either contiguous or noncontiguous time intervals as

$$T = \sum_{i=1}^{n} T_i \quad [31]$$

Of course, the restrictions of equations [25] and [26] still apply and, furthermore, because of the multiple T_i, we will now have more than one t_0, i.e., t_{0i} where i = 1, 2, . . . n.

In this example the T_i are contiguous and, at first, it may appear that subdividing T is unnecessary. However, to perform the integration of equation [23] when the pdf's are stationary and periodic requires that the total integration interval be subdivided into time intervals commensurate with the $p_i(S)$ of equation [17], i.e., the month.

Within each T_i interval, equation [26] is applicable, i.e.,

$$W = W(A, C, t_{0i}, T_i, \tau, P) \quad [32]$$

and the statistical average for the total interval T can be shown to be

$$W_\tau = \sum_{i=1}^{n} \left(\frac{T_i}{T}\right) W(A, C, t_{0i}, T_i, \tau, P) \quad [33]$$

i.e., the average crop rotation period soil is the time weighted sum of each monthly W. W_τ is a shorthand notation for

$$W_\tau \triangleq W(A, C, \tau, \tau, \tau, P) \quad [34]$$

Substitution of equations [10] and [23] into [33] yields

$$W_\tau = \frac{1}{AT} \left\{ \sum_{i=1}^{12} \int_{t_{0i}-T_i}^{t_{0i}} \int_{-\infty}^{\infty} \dot{m}\, p_i \, dS\, dt + \sum_{i=13}^{24} \int_{t_{0i}-T_i}^{t_{0i}} \int_{-\infty}^{\infty} \dot{m}\, p_{i-12}\, dS\, dt \right\} \quad [35]$$

Example III

The average monthly soil erosion, say for the month of March, for a crop cycle of 4 years implies that t_{0i} = March 31 for 4 successive years, T_i = 31 days for i = 1, 2, 3, 4, and τ = 4 years. This case is derivable from the results of Example II, i.e., equation [35], when it is noted that the T_i would represent a subset of a full 4 years of monthly W computations. The equation is determined by conceptually expanding equation [35] to four summations, one for each year of the crop rotation, and then selecting only the four March integrals, i.e.,

$$W(A, C, \vec{t_o}, 93 \text{ days}, 4 \text{ years}) = \frac{1}{AT} \sum_{i=1}^{4} \int_{t_{0i}-T_i}^{t_{0i}} \int_{-\infty}^{\infty} \dot{m}\; p_3\, dS\, dt \quad [36]$$

Here we denote the four t_{0i} as a vector, $\vec{t_0}$

From this example it can be seen how to construct the appropriate equation to compute W for *any* desired subinterval of τ.

CONCLUSIONS

1. Soil erosion, w, as a measure of the erosion process for finite time and space intervals is an arithmetic average of f_z either into the past or future. Computation of w requires that all of its independent variables must be specified functions, i.e., deterministic.
2. Average soil erosion, W, however is a measure of the future erosion process, i.e., the mean of the distribution of all future w's. As such, W requires that its stochastic variables be specified as a joint pdf, and the deterministic variables must be periodic. W requires that the difference between T and τ must be considered.
3. To properly describe W requires that T must be included in the nomenclature, e.g., average March wind erosion or average yearly wind erosion. Since $T = \tau$ is the most frequent case for W, a reasonable abbreviation for "the average crop rotation period soil loss" would be "the average soil erosion".
4. The units of w and W should be t/(ha·yr) and should not depend on the intervals T or τ, e.g., if T is all Marches in $\tau = 3$ years, then W is the average March wind erosion expressed in units of t/(ha·yr). Consistency of units allows for ease of comparison between W's.
5. Proof of the validity of the equation for w is possible from experiments, with T in the order of hours or days. Proof of W is practically (if not theoretically) impossible, due to the time required to obtain an adequate number of samples of w, e.g., 30 or 40 τ intervals. It appears that we must content ourselves with a proof of w and an assumption that W is valid.

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TABLE 1. NOTATION. M, L, AND T AS DIMENSIONS REFER TO MASS, LENGTH, AND TIME

Symbol	Definition and dimensions
A	area of a surface, L^2
C	the perimeter of A, L
D	the set of all deterministic variables, see equation [11]
E_c	potential average annual soil loss, $M\,L^{-2}\,T^{-1}$
E(·)	expected value of a random variable, dimensions depend on the random variable
f	soil flux vector, $M\,L^{-2}\,T^{-1}$
g	probability density function of m
I	soil erodibility, $M\,L^{-2}\,T^{-1}$
J	the set of surface conditions indicated in equation [8]
K	soil ridge roughness, dimensionless
M	Precipitation, dimensions unknown
m	soil loss, the mass that has flowed from surface A for a given interval of time, M
$\dot{m}$	the soil mass flow rate from surface A, $M\,T^{-1}$
n	the number of trials or upper limit of the index i, dimensionless
P	parameters of S and D, dimensions unknown
p	joint probability density function of S
S	the set of all stochastic variables, see equation [12]
T	the soil loss accounting interval, see Fig. 1, T
t	time, T
t_0	the upper limit of the interval T, see Fig. 1, T
U	windspeed, $L\,T^{-1}$
V	equivalent quantity of vegetative cover, $M\,L^{-2}$
W	the statistical mean of w or average soil erosion, $M\,L^{-2}\,T^{-1}$
W_τ	see equations [33] and [34], $M\,L^{-2}\,T^{-1}$
w	soil erosion, see equation [9], $m\,L^{-2}\,T^{-1}$
x	distance along the x axis, L
y	distance along the y axis, L
z	distance along the z axis, L
β	wind angle, the angle of the wind relative to the positive x axis, counter-clockwise positive, dimensionless
τ	time duration of a trial, generally the crop rotation period, see Fig. 1, T
Subscripts	
i	index, 1, 2, 3, . . . various time intervals
z	z component
Superscripts and other symbols	
$\triangleq$	defined
^	implies that the variable is a function of time and space
$\rightarrow$	implies that the variable is a vector