Using Two Sieves to Characterize Dry Soil Aggregate Size Distribution

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ABSTRACT

SURFACE soil aggregate size distribution affects many facets of agriculture from wind erosion susceptibility to seedbed suitability. The log-normal distribution generally provides a good description of aggregated soil size distribution. Unfortunately, other measures of aggregation have often been adopted, because they were perceived as easier to apply. In this study, a method to calculate the geometric mean diameter, D_g, and geometric standard deviation from two sieve cuts was developed for log-normal distributions. Results from 10 soil samples using the two-sieve procedure were compared to results from the same samples using multiple seive cuts. The multiple sieve data were analyzed using both a traditional graphical and a linearized least-squares procedure to predict D_g and percentage aggregates greater than 0.84 mm.

All the methods gave nearly equal size distribution parameters. The two-sieve procedure is least laborious but does not permit easy detection of samples that deviate from a log-normal distribution.

INTRODUCTION

The size distribution of dry surface soil aggregates affects many facets of agriculture from wind erosion susceptibility (Chepil, 1950, 1953) to seedbed suitability (Hadas and Russo, 1974; Schneider and Gupta, 1985). Gardner (1956) demonstrated that the log-normal distribution provided a good description of the aggregate size distribution on many soils. Kemper and Chepil (1956) extolled the virtue of summarizing aggregate size distribution data with the paramters geometric mean diameter, D_g, and geometric standard deviation, σ_g, but did not recommend this method for general use because of the extensive work to determine the parameters. Consequently, less meaningful measures of aggregate size distribution have often been adopted. The purpose of this research was to develop a less laborious method for determining the log-normal distribution parameters for summarizing soil aggregate size distribution data.

THEORY

For aggregates that are size distributed log-normally, the mass fraction, P_i, of aggregates whose diameters are greater or less than some diameter, D_i, may be represented by use of the error function of the normal distribution. This function associated with the normal curve is

$$\frac{1}{2}\operatorname{erf}\left(\frac{t}{\sqrt{2}}\right) = \int_0^t \phi(x)dx \quad \ldots\ldots [1]$$

where the right hand of equation [1] is the area integral of the normal probability curve (Hodgman et al., 1957, p. 237). The error function has the following properties:

$$\operatorname{erf}(-Z) = -\operatorname{erf}(Z) \quad \ldots\ldots [2]$$

$$\operatorname{erf}(\infty) = 1 \quad \ldots\ldots [3]$$

$$\operatorname{erf}(0) = 0 \quad \ldots\ldots [4]$$

and is defined by

$$\operatorname{erf}(Z) = \frac{2}{\sqrt{\pi}} \int_0^Z e^{-t^2}\, dt \quad \ldots\ldots [5]$$

where t is a dummy variable of integration (Gautschi, 1965). In our application of the error function

$$Z_i = (\ln(D_i/D_g))/(\sqrt{2}\ \ln \sigma_g) \quad \ldots\ldots [6]$$

D_g and σ_g are geometric mean diameter and geometric standard deviation, respectively.

The mass fraction of aggregates, P_i, whose diamters are greater than D_i, is:

$$P_i = 0.5 - \operatorname{erf}(Z_i)/2 \quad \ldots\ldots [7]$$

and the mass fraction of aggregates P_i, whose diameters are less than D_i, is:

$$p_i = 0.5 + \operatorname{erf}(Z_i)/2 \quad \ldots\ldots [8]$$

where Z_i is defined by equation [6].

Given P_1 and P_2 from two seive cuts, one can solve equation [7] to give

$$\operatorname{erf}(Z_1) = (1 - 2P_1) \quad \ldots\ldots [9]$$

$$\operatorname{erf}(Z_2) = (1 - 2P_2) \quad \ldots\ldots [10]$$

The $\operatorname{erf}(Z_1)$ and erf (Z_2) can be calculated directly from

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equations [9] and [10]. Then Z_1 and Z_2 can be calculated from an iterative computer procedure. Finally, substituting Z_1, D_1 and Z_2, D_2 into equation [6] gives two equations with two unknowns D_g and σ_g. Eliminating variables gives

$$D_g = \text{Exp}\left[\frac{Z_1 \ln D_2 - Z_2 \ln D_1}{Z_1 - Z_2}\right] \quad \text{[11]}$$

and

$$\ln \sigma_g = \ln(D_1/D_g)/(\sqrt{2}\, Z_1) \quad \text{[12]}$$

Given D_g and σ_g for a sample of aggregates, one can easily compute other parameters of interest. For example, the mass fraction greater than some arbitrary diamter D_3 can now be calculated by solving equations [6] and [7], respectively.

One may also determine the distribution parameters D_g and σ_g from a more complete sieving obtained by sieving the sample into several cuts. Since the plot of log-normally distributed data form a straight line on a log-probability graph, the results of sieving can be fit by the method of least squares to an equation of the familiar form

$$Y = a + b\,X \quad \text{[13]}$$

where Y is log D_i, a is intercept, b is slope, and X is a linearized probability scale. A procedure to linearize the scale is demonstrated later.

PROCEDURE

In order to compare the two-sieve method to other methods of finding the aggregate size distribution, soil sieving data were obtained from a joint SCS and ARS investigation of soil erodibility of the soils in the Texas High Plains. Three to 5 kg samples of Pullman clay loam (fine, mixed, thermic Torrecertic Paleustalfs) and Amarillo loamy fine sand (fine-loamy, mixed, thermic Aridic Paleustalfs) were collected periodically from the surface 3 cm, oven dried at 105°C, and sieved with a standard compact rotary sieve (Chepil, 1952). The sieve sizes were 0.42, 0.84, 2.38, 6.4, and 12.7 mm. Geometric mean diameter and mass fraction of sample greater than 0.84 mm were determined by three methods: (a) graphically, (b) computed from log-normal distribution parameters that were determined from two sieve cuts (0.42 and 6.4 mm), and (c) computed using more complete sieving.

Graphical Method

The graphical determination was accomplished by plotting aggregate diameter vs percent by weight greater than the stated diameter on log-probability graph paper. The geometric mean diameter on a mass basis is defined as the diameter at which 50% of the material by weight is greater than and 50% is smaller than D_g and the geometric standard deviation is the ratio of sizes (Irani and Callis, 1963):

$$\sigma_g = \frac{\text{aggregate size at 50\% oversize}}{\text{aggregate size at 84.1\% oversize}}$$

$$= \frac{\text{aggregate size at 15.9\% oversize}}{\text{aggregate size at 50\% oversize}} \quad \text{[14]}$$

Two-Sieve Method

The mass fraction of aggregates whose diameters (D_1, D_2) were greater than 0.42 and 6.4 mm were substituted for P_1 and P_2 into equations [9] and [10], and erf(Z_1) and erf(Z_2) were calculated. Using erf(Z_1) and erf(Z_2) and an interactive computer procedure with the computer compiler's error function subroutine, we computed Z_1 and Z_2. Z_1, D_1 and Z_2, D_2 were substituted into equation [11] and D_g was calculated. Ln σ_g was also calculated from equation [12].

With the distribution parameters D_g and ln σ_g now known, we used equation [6] and equation [7] to calculate the mass fraction of aggregates greater than 0.84 mm in each of 10 data sets of the Pullman and Amarillo soils.

Multisieve Method

The third method required a transformation of the probability scale into a linear one. The distance from 0.1 and other probabilities to 99.9 on probability graph paper from normal distributions was measured in arbitrary units. This data set of probability vs SCALE at 50% and 15.9% probabilities were determined to give mean and standard deviation of 15.75 and 5.2, respectively.

The error function associated with the normal probability integral, equation [1], was used to obtain data sets of aggregate diameter and SCALE. These data obtained and the geometric mean diameter was determined in several steps:

Step 1. The mass fraction P_i greater than each of the four smallest sieve sizes, D_i, was calculated from sieving data (Table 1)

Step 2. Using P_i from Step 1, equation [8] was solved with an interactive routine as in Method 2 to obtain the value of the argument of the error function, Z_i. For this case,

$$Z_i = (S_i - \bar{S})/(\sqrt{2}\,\sigma \quad \text{[15]}$$

where S_i is the value of SCALE corresponding to P_i; $\bar{S}$ and σ are the mean (15.75) and standard deviation (5.2) of SCALE distribution.

Step 3. Equation [15] was solved for S_i corresponding to each P_i from Step 1, which along with sieving results yields data sets of (D_i, S_i).

Step 4. The least squares fit the log D_i vs S_i was determined for the model of equation [13].

Step 5. Each of the regression equations from Step 4 was used to calculate log D_i at S = 15.75. The antilog was then calculated to give the geometric mean diameter for each aggregate sample.

Step 6. Each of the regression equations from Step 4 was solved for S_i at an aggregate diameter equal to 0.84 mm to give the value of SCALE corresponding to an aggregate diameter of 0.84 mm.

Step 7. Z_i was calculated from equation [15] for each S_i calculated in Step 6.

Step 8. Z_i from Step 7 was substituted into equation [8] to find the mass fraction of the sample having aggregates greater than 0.84 mm.

RESULTS AND DISCUSSION

The aggregate size distributions of Amarillo lfs and Pullman cl as determined from dry sieving on five sampling dates are given in Table 1. Table 2 shows

TABLE 1. AGGREGATE SIZE DISTRIBUTION OF AMARILLO LOAMY FINE SAND (FINE-LOAMY, MIXED, THERMIC ARIDIC PALEUSTALF), BAILY CO., TEXAS; AND PULLMAN CLAY LOAM (FINE, MIXED, THERMIC TORRERTIC PALEUSTOLL), CARSON CO., TEXAS

Sample soil/Date	Percent greater than indicated diameter, mm				
	0.42	0.84	2.38	6.4	12.7
Amarillo					
08 Dec. 1981	50.2	46.5	40.0	26.1	4.6
16 Mar. 1983	23.0	17.0	12.9	7.4	0.5
24 Aug. 1983	61.1	57.5	52.0	40.1	14.3
12 Oct. 1983	83.4	34.7	29.1	17.3	1.7
04 Jan. 1984	29.6	25.5	20.0	9.3	0.7
Pullman					
31 Mar. 1983	88.1	82.9	75.8	65.3	36.0
12 Apr. 1983	69.6	58.4	42.1	31.2	11.9
01 Aug. 1983	76.1	71.0	62.4	48.0	20.4
05 Mar. 1984	58.5	46.6	32.5	22.3	6.7
04 Mar. 1985	48.8	36.3	24.1	16.3	5.5

results of various steps in the multisieve method.

For most samples, the aggregate sizes were distributed log-normally, except for the largest size as indicated by the plot of Fig. 1. The plots of other data sets were similar to those of Fig. 1, with the 12.7 mm aggregates deviating from a straight line. Occasionally, the tailing off started with the 6.4 mm aggregates, as seen in one sample in Fig. 1.

All three methods agreed reasonably well for determining D_g (Table 3). The coefficient of determination for linear regression between methods was 0.97 and above (Table 4). Calculation of the confidence intervals for the intercepts (a) and slopes (b) showed that in all cases the hypotheses that a=0 and b=1 could not be rejected at the 95% confidence level. Much of the variation was attributed to one data set (Fig. 2). The > 6.4 mm size fraction from the 4 January 1984 sampling of the Amarillo deviated from a straight line on a log-normal plot. When those data were deleted, the coefficients of determination for D_g were greater than 0.99.

The percent of aggregates > 0.84 mm as calculated using the distribution parameters agreed well with the sieved values (Table 3). The coefficients of determination for linear regression between methods were equal to or greater than 0.99 (Table 4).

The results of this experiment indicate that graphical, two-sieve, and multiple-sieve computational methods all

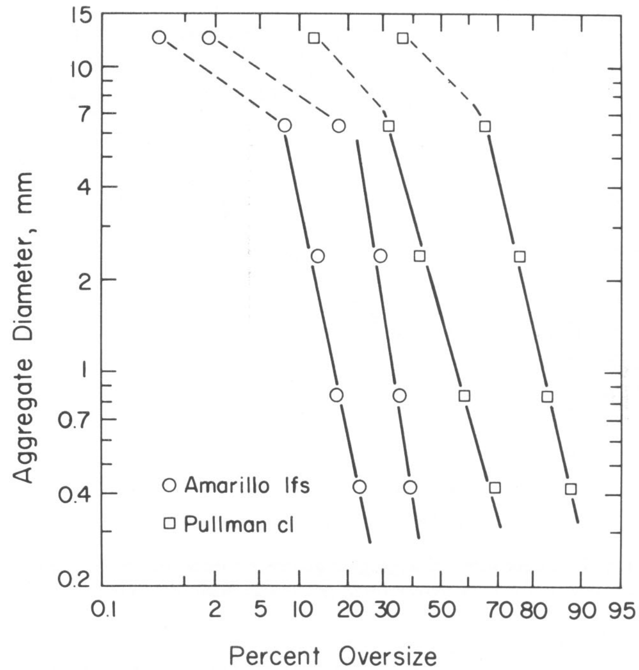

Fig. 1—Aggregate size distribution of Amarillo loamy fine sand and Pullman clay loam as determined by dry sieving at two different sampling dates for each soil.

can be used for determining aggregate size distribution parameters. All three methods are contingent upon soil-aggregate size being log-normally distributed. A deviation from a log-normal distribution would be detected visually by plotting multiple sieve cuts in the graphical method or by a low r^2 as in Table 4 for a least squares fit to sieved data, whereas, it would go undetected when using only two sieve cuts for either a graphical or computional determination of aggregate size distribution parameters. Although past experience has shown that soil aggregates' size is generally log-normally distributed, a formal statistical test such as a chi-square goodness-of-fit test can be applied to multiple-sieve data to test the hypotheses that the data fit the log-normal distribution. In aggregated soil samples, only the extreme tails of the size distribution will often deviate from log-normality. This may be caused by tillge operations limiting the upper aggregate sizes and the primary particle size distribution limiting frequency of the smallest sizes. If the extreme tails of the distribution

TABLE 2. LINEAR REGRESSION AND DETERMINATION COEFFICIENTS OF STEP FOUR IN MULTI-SIEVE METHOD, AND RESULTS OF INTERMEDIATE CALCULATIONS

Sample soil/Date	Regression coefficients a	b	r^2	log D_i*	S_i†	Z_i‡
Amarillo						
08 Dec. 1981	5.037	-0.335	0.941	-0.239	15.3	-0.067
16 Mar. 1983	3.534	-0.329	0.980	-1.648	11.0	-0.650
24 Aug. 1983	6.839	-0.414	0.950	0.319	16.7	0.129
12 Oct. 1983	4.634	-0.344	0.932	-0.784	13.7	-0.280
04 Jan. 1984	3.366	-0.279	0.930	-1.028	12.3	-0.464
Pullman						
31 Mar. 1983	6.050	-0.294	0.997	1.420	20.8	0.692
12 Apr. 1983	3.725	-0.224	0.994	0.197	17.0	0.166
01 Aug. 1983	5.553	-0.303	0.982	0.781	18.6	0.385
05 Mar. 1984	3.522	-0.233	0.995	-0.148	15.4	-0.042
04 Mar. 1985	3.285	-0.238	0.990	-0.464	14.1	-0.222

*, †, ‡, calculated in steps 5, 6, 7, respectively.

TABLE 3. GEOMETRIC MEAN DIAMETER AND PERCENT GREATER THAN 0.84 mm COMPARED FOR THREE METHODS OF DETERMINATION

Sample soil/Data	Graphical D_g	Graphical >0.84*	Two sieve D_g	Two sieve >0.84	Multiple sieve D_g†	Multiple sieve >0.84‡
	mm	%	mm	%	mm	%
Amarillo						
08 Dec. 1981	0.5	46.5	0.43	43.7	0.57	46.3
16 Mar. 1983	0.02	17.0	0.03	17.9	0.02	17.9
24 Aug. 1983	2.0	57.5	1.77	55.8	2.08	57.3
12 Oct. 1983	0.1	34.7	0.12	32.2	0.16	34.6
04 Jan. 1984	0.02	25.5	0.07	23.1	0.09	25.6
Pullman						
31 Mar. 1983	30.0	82.9	24.9	83.7	26.2	83.6
12 Apr. 1983	1.5	58.4	1.69	60.2	1.56	59.3
01 Aug. 1983	8.0	71.0	5.35	69.9	6.04	70.7
05 Mar. 1984	0.7	46.6	0.76	48.6	0.72	47.6
04 Mar. 1985	0.38	36.3	0.39	39.3	0.35	37.7

*Sieved Value; †, ‡ results of steps 5 and 8, respectively.

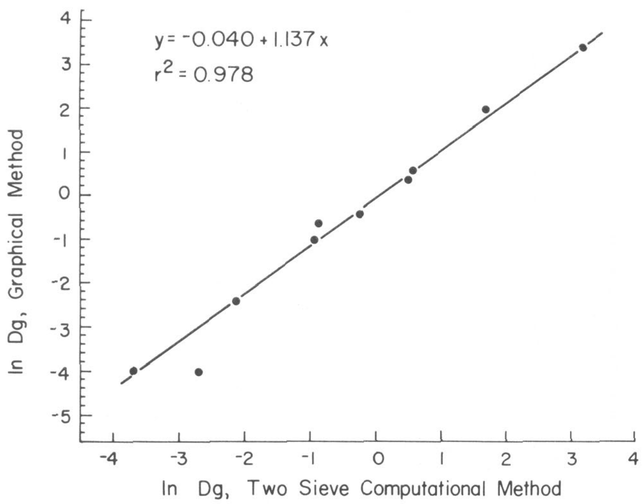

Fig. 2—Logarithms of aggregate geometric mean diameter compared for two methods of determining geometric mean diameter.

are important to the application planned for the data, one can fit a 3 or 4 parameter log-normal distribution to multiple sieve cuts using nonlinear regression techniques (Raabe, 1978).

When using two sieves, we recommend that sieve sizes be selected so that at least 10% of the sample is collected on the larger sieve and at least 10% of the sample passes through the smaller sieve. Sieves Number 40 and Number 3, with openings of 0.42 and 6.35 mm, respectively, meet these criteria for many aggregated soils.

Ease and simplicity of the computational procedures, especially the two-sieve method, should overcome the hesitancy to use log-normal distribution function parameters for summarizing soil aggregate size distribution data. A short FORTRAN computer program is available from the authors which will rapidly compute D_g, σ_g, and percentage mass greater than some user selected aggregate diameter for any number of soil samples, given two sieve cuts per sample as input.

TABLE 4. LINEAR REGRESSION AND DETERMINATION COEFFICIENTS BETWEEN METHODS FOR DETERMINING GEOMETRIC DIAMETER, D_g, AND PERCENT OF AGGREGATES GREATER THAN 0.84 mm

	Linear regression coefficients			
Variable	Model*	a	b	r^2
D_g	ln M1 vs ln M2	-0.040	1.137	0.978
D_g	ln M1 vs ln M3	-0.135	1.136	0.969
D_g	ln M2 vs ln M3	0.082	1.002	0.994
% > 0.84	M1 vs M2	1.068	0.982	0.989
% > 0.84	M1 vs M3	-0.632	1.004	0.999
% > 0.44	M2 vs M3	-1.351	1.015	0.994

*M1, M2, and M3 are graphical, two-sieve, and multiple sieve methods, respectively.

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