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MODELING TILLAGE ACTIONS ON SOIL AGGREGATES

by

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SUMMARY:

A Markov chain based two-parameter stochastic model has been developed to predict tillage-induced crushing of soil aggregates. Model parameters have been identified for several tillage operations on two soil types from field data. The simulation results suggest that the crushing model can predict tillage effects on soil aggregate size distribution reasonably well with average prediction errors within 3% for the limited cases available for verification.

KEYWORDS:

stochastic models, aggregate size distribution, tillage, crushing, simulation

Abstract[*]

A Markov chain based two-parameter model has been developed to model the tillage-induced crushing of soil aggregates. Model parameters were identified for several tillage operations on two soil types with a downhill simplex multidimensional minimization approach. The simulation results suggest that the crushing model can predict tillage-induced aggregate crushing reasonably well. The average prediction errors are within 3 percent for the limited cases of verification. This study indicates that the stochastic simulation is better than the conventional deterministic method in estimating the tillage effects on soil aggregate size distribution because of the apparent randomness of the variability in the field data.

Introduction

The Wind Erosion Prediction System (WEPS), presently being developed by the Agricultural Research Service, USDA (Hagen, 1991), need the service of a TILLAGE submodel for simulating the effects of various tillage and management operations performed on farm soils. One of the major tillage actions is crushing or breaking of soil aggregates (clods) as pointed out by Cole (1988). The objective of this study is to model the crushing effect on soil aggregates under different soil conditions and tillage practices. The specific task of this study is to develop a simulation model based on field collected pre-tillage and post-tillage aggregate size distribution samples. The model will then be incorporated into the TILLAGE submodel, where post-tillage aggregate size distribution values will be predicted from pre-tillage aggregate size distribution, tillage operation being performed, soil type, and possibly other factors.

A deterministic model[1], Eq.(1), which was originally used for modeling solid grinding processes (Austin, 1971/1972) based on the conservation of mass principle, was employed to model the crushing process.

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[1] Deterministic model means that all the components of the model are deterministic.

$$w_1[i] = (1 - s_i) w_0[i] + \sum_{k=i+1}^{N} (B_{k,i+1} - B_{k,i}) s_k w_0[k] \tag{1}$$

where: $w_0[i]$ is the initial mass aggregate size fraction distribution

$w_1[i]$ is the final mass aggregate size fraction distribution

$B_{k,i}$ is the *cumulative distribution function* (mass fraction of material broken from size class k which falls into smaller size classes than size interval i)

s_i and s_k are the selected fractions of size interval i and k, respectively, for breakage

N is the total number of sieve sizes

In words, Eq.(1) can be stated as "the mass of material in size class i after one stage of tillage equals the sum of material broken into size class i from larger size classes plus the original material in size class i minus the material broken out of size class i". It is obvious that the success of Eq.(1) relies on how well we can estimate $B_{k,i}$ and s_i for different tillage operations and soil conditions, or in other words, how sensitive the parameters in functions $B_{k,i}$ and s_i responds to various situations. Many possible functional forms for both $B_{k,i}$ and s_i as suggested by Austin (1984) were tried with back-calculation parameter estimation procedures used by Gupta (1981). But the results did not suggest a clear relationship between the parameters estimated and soil conditions because: a) it is hard to find two appropriate functional forms for $B_{k,i}$ and s_i simultaneously, and b) the model, Eq.(1), is too sensitive to the variability (noise) existing in the field data.

As a result of the difficulties with the deterministic model, a stochastic approach was pursued to model the crushing process because: a) significant variance in the field data existed which could be treated as a random process, and b) a unified treatment would enable us to avoid the complications encountered with a separate $B_{k,i}$ and s_i. This report describes some of the efforts in that stochastic model development.

Experimental Data Sets

The aggregate size distribution (ASD) data sets used in this study were obtained from several experimental field studies, some of which have been published (Tangie, et al, 1990; Ambe, 1991; and Wagner, et al, 1991). All of the experiments were conducted on two soils (Table 1): Kimo silty clay loam (clayey over loamy, montmorillonitic, mesic Aquic Hapludolls) and Eudora silt loam (coarse-silty, mixed, mesic Fluventic Hapludolls) at the Kansas River Valley Experiment Field near Topeka, Kansas. Individual objectives and experimental designs of these field studies varied, however each of these experiments contained pre- and post-tillage ASD measurements of which some were suitable for use in the development of the stochastic aggregate crushing model.

The quantity and number of replications of ASD samples used varied among the experiments but all were collected and processed in the same manner. ASD samples (approx. 10 kg) were collected from the first 15 cm of soil or from within the entire prior tillage depth for consolidated and unconsolidated pre-tillage soil conditions respectively. Post-tillage ASD samples were obtained from within the resulting tillage tool processing depth. These samples were extracted at randomly selected locations (between wheel tracks) in each plot using a 30 x 23 cm flat square-cornered shovel, as described by Chepil (1962), and placed in 46 x 30 x 6 cm plastic tubs. All aggregate size distribution samples were air-dried in a greenhouse prior to sieving with a modified combined rotary sieve (Lyles et al., 1970).

Suitable ASD data sets were available for a variety of tillage implements although the size of the data sets varied among the tillage tools with some implements having multiple data sets for both soils to single data sets on only one soil. All speeds and depths were typical for each respective tillage operation. Those tillage implements were:

1. offset disk - 45 cm dia. blades with a 30 cm inter-disk spacing.
2. point chisel - two ranks of 36 cm deep curved shanks with an inter-tool rank spacing of 60 cm.
3. springtooth cultivator - three ranks with an inter-tool rank spacing of 45 cm.
4. rotary tiller - typical garden tractor powered rotary tiller with a blade radius of 16 cm.

Model Description and Identification

The stochastic model[2] for the crushing process is a Markov[3] chain model (Bhat, 1984), which can be stated as follows in the context of the soil aggregate crushing process:

> A soil aggregate is assumed to consist of many particles with each having an infinitesimal volume and a unit mass. The soil particles can only travel downward from a larger aggregate size class to smaller aggregate size classes after each tillage pass (crushing of an aggregate). If a size class is called a "state", then the transition of soil particles from one state to another can be treated as a completely random event. A probability matrix, *P[i,j]*, can be constructed for all possible transitions occurring in the soil when its aggregate size distribution (mass fractions across different size classes) shifts or transfers from $w_0[i]$ to $w_1[k]_{(0\ to\ i\text{-}1)}$ after one tillage pass. *P[i,j]*, often called a *transition matrix*, maintains the properties of a Markov chain and does not change with the number of tillage passes performed but depends on the type of tillage and the specific soil conditions.

Mathematically, the Markov chain based crushing model has a very simple form:

$$w_1[i]_{(1\times n)} = w_0[i]_{(1\times n)} P[i,j]_{(n\times n)} \tag{2}$$

The effectiveness of the model relies on how accurately the transition matrix *P[i,j]* can be estimated. According to the model statement, the transition matrix, *P[i,j]*, can be generalized as a lower triangular matrix, where the states with smaller index values correspond to the smaller aggregate size classes (size intervals) and vice versa.

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[2] Stochastic model is a model which has at least one component which will be treated as exhibiting random behavior.

[3] A Markov process is one in which the next "state" is dependent only on the present "state" and is independent of any previous "states".

$$P[i,j] = \begin{pmatrix} p_{11} & 0 & & \cdots & & 0 \\ p_{21} & p_{22} & 0 & \cdots & & 0 \\ \vdots & & & & & \\ \vdots & & & & & \\ p_{i1} & \cdots & p_{ij} & \cdots & p_{ii} \; 0 \; \cdots & 0 \\ \vdots & & & & & \\ p_{n1} & p_{n2} & p_{n3} & \cdots & p_{nn-1} & p_{nn} \end{pmatrix}_{n \times n} \tag{3}$$

Since it is almost impossible to estimate each transition probability, p_{ij}, on a one by one basis, it is assumed that the p_{ij} observes a binomial distribution[4] as shown in Eq.(4), since the binomial distribution is one of the simplest and most profound discrete probability distribution functions.

$$p_{ij} = \binom{i-1}{j-1} p_i^{j-1}(1-p_i)^{i-j} \qquad i = 1,2,...,n\ ,\ \ j = 1,2,.....,i \tag{4}$$

In Eq.(4) p_i is defined as the *probability function of breakage,* which has a value within the interval [0,1] and can be generally expressed as an algebraic function of sieve size x_i and a number of parameters α_1, α_2,...,α_m.

$$p_i = f(x_i, \alpha_1, \alpha_2, ..., \alpha_m) \tag{5}$$

The probability function of breakage, p_i, reflects how much breaking is occurring in the aggregate size class *i.* The larger p_i is, the smaller the percentage of soil aggregates of size class *i* that will break into smaller size classes. If $p_i = 1$, then no aggregates of size class *i* are being broken down and if $p_i = 0$, then all of the aggregates in size class *i* are being broken down into smaller size classes.

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[4] It is the probability distribution of Bernoulli trials, which are repeated independent trials. Each trial has two possible outcomes and the corresponding probability remains the same for all trials.

It is presumed intuitively that p_i is related to the tillage tool, the soil conditions and the sieve cut sizes used in measuring $w_0[i]$ and $w_1[i]$. Therefore, the α_i parameters in Eq.(5) are expected to be functions of those conditions.

$$p_i = f(x_i, \alpha, \beta) \tag{6}$$

In this analysis, the focus is on a two parameter representation of a breakage probability function such as Eq.(6), primarily because: 1) it is complicated to identify multiple parameters, and 2) the size of the data file for model parameterization is small (8 pairs of data points as shown in Table 2 for an eight-cut rotary sieve). Therefore, two parameters were deemed adequate to reflect the variability of the data.

The model identification includes finding a suitable p_i function and then searching for the parameters of the function for different tillage tools and soil conditions. Several functional representations of p_i were identified. Initial study suggests that four functions are most promising. They are:

$$p_i = \alpha\left(1 - \beta\frac{x_i}{x_{\max}}\right) \tag{7}$$

$$p_i = \alpha\left(\frac{x_i}{x_{\max}}\right)^{\beta} \tag{8}$$

$$p_i = 1.0 - \exp\left(-\alpha + \beta\frac{x_i}{x_{\max}}\right) \tag{9}$$

$$p_i = \frac{1.0}{1 + \exp\left(-\alpha + \beta\frac{x_i}{x_{\max}}\right)} \tag{10}$$

where: x_i's are individual sieve sizes and x_{max} is the maximum sieve size.

α and β are the parameters to be determined.

A back-calculation procedure is used to estimate α and ß in the above equations based on known $w_0[i]$ and $w_1[i]$. It is a multi-dimensional minimization of the target function:

$$\varepsilon = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(w_1[i] - \sum_{j=i}^{n} w_0[j][p_{ji}]\right)^2} \tag{11}$$

Two types of multi-dimensional minimization algorithms are used in the parameter identification. One is the gradient-based method which requires calculation of derivatives for the function (Press et al, 1988). It was found that the gradient based-methods converge slowly and are very sensitive to the initial conditions. Another type of minimization algorithms are gradient-free methods such as the Powell minimization and the downhill simplex method. It was found that the gradient-free methods gave almost identical results and showed less sensitivity to the initial conditions. Most of the calculations were carried out with the downhill simplex method. A computer program was written in the C language and based on code published by Press et al (1988).

Data used for model parameterization were extracted from experimental field data. The data were grouped into the format as shown in Table 1 after computing the mass percentage distributions, removing data sets containing apparent errors, and averaging multiple observations for each measurement. Because of field data collection problems and the randomness associated with the tillage operation and field conditions, there were still relatively large variances in the data sets.

Results and Discussion

Model identification was carried out with data collected from two soil types, a silt loam and silty clay loam soil (Table 1). Four different tillage implements were used: an offset disk, rotary tiller, point chisel and springtooth cultivator.

The most suitable functional representation for p_i is found to be the form of Eq.10 as shown in Eq.12.

$$p_i = \frac{1.0}{1 + \exp\left(-\alpha + \beta \frac{gmd_i}{gmd_{max}}\right)} \tag{12}$$

where[5]: gmd_i are geometric means of x_{i-1} and x_i (geometric mean dia. of aggregates in each size class)

gmd_{max} is geometric mean of x_n and x_{n+1} (geometric mean dia. of aggregates in largest size class)

Parameter α reflects the breakage of all soil aggregates regardless of size. As α decreases, the percentage of soil aggregates breaking increases. Parameter ß reflects the unevenness of breakage among aggregates in different size classes. Large ß values means that crushing mainly affects the large soil aggregates.

The parameters in the model represented by Eq.(2), (3), (4) and (12) were estimated for four tillage tools using the back calculation procedure. The results are listed in Table 3. Although the parameters in Table 3 are derived from only a few sets of field data, they give very good indications of how much crushing each tillage tool causes. Based on the α values for the silt loam, the tiller produces the most overall crushing and the cultivator produces the least. The disk has more crushing of larger aggregates because of its relative large ß value. For the silty clay loam, the tiller still generates the most crushing while the disk and cultivator exhibit strong effects on large aggregates.

To judge how much crushing is caused by a tillage implement based upon its two parameters, the following rules of thumb can be applied:

1. See whether α is small (less than 1.5). If so, then the tillage tool will be crushing or breaking down a large percentage of the aggregates.

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[5] For a rotary sieve of n sieves, the x_0 and x_{n+1} are arbitrary minimum and maximum aggregate sizes assumed to exist in the data. The values used in this analysis were 0.10 mm and 152.4 mm respectively as shown in Table 2.

2. If α is not small, then see if β is equal to or larger than α. If so, then the tillage tool will be breaking down a high percentage of the larger aggregates.
3. If α is large and β is small, then the tillage tool will produce little crushing or breaking down of soil aggregates.

Relatively large standard deviations shown in Table 3 encountered in the parameter identification processes are caused by the variability in the field data. Major problems can exist in collecting valid field data. Some are: a) measuring the ASD of compacted soil, b) obtaining accurate estimates of field ASD values of sandy soils because of aggregate susceptibility to damage during the rotary sieving process, and c) the effects of water content.

With the parameters identified, the crushing model was further verified with limited data sets from other field experiments. Figure 1 to Figure 4 show four of the simulation results of the offset disk on two types of soil. It can be seen that the offset disk causes significant breakup of large clods of silty clay soil (Figure 1) and causes little breakup of same soil without large clods (Figure 2). The crushing induced by the offset disk on sandy soil, i.e., silt loam, has different characteristics from the crushing of clay soil as shown in Figure 3 and 4, where the whole ASD curves shift downward after the tillage. Figure 3 and 4 also suggest that the soil moisture content seems not affecting the crushing significantly. The four plots show that the model can predict the disk-induced crushing processes reasonably well. As a matter of fact, the prediction errors, defined as the average error across all the size classes, are within 3 percent for all of the verification cases. However, the crushing model needs to be further verified with data from other tillage experiments.

The stochastic crushing model can also be used in reverse by reversing the transition matrix *P[i,j]*, to estimate pre-tillage aggregate mass fraction distributions based on the measurement of the post-tillage distribution. Another possible application is modeling a series of crushing events by multiplying the transition matrix associated with each tillage tool together.

Summary and Conclusions

This study can be summarized as follows:

1. The deterministic model represented by Eq.(1) cannot simulate tillage-induced crushing accurately because of difficulties associated with parameter identification due to the large variability in the field data.

2. The stochastic model appears suitable for modeling the crushing process when described as a random process. The Markov chain based two-parameter stochastic crushing model appears to give consistent and fairly accurate estimations of disk-induced crushing based on the limited data analyzed. The simulation errors are within 3 percent.

3. More tillage data, with other implements under various soils and soil conditions are needed to extend the application of this modeling approach.

4. To increase the precision of parameter identification, the field data measurement and collection procedures will require improvement to reduce the variability in pre- and post-tillage aggregate size distribution data.

5. Although the stochastic crushing model has successfully simulated tillage-induced crushing on two types of soil (assuming one soil condition) and has given a consistent estimation of the parameters involved, there is still a major challenge ahead to parameterize the model for various soil and tillage conditions. This will require a large amount of field data from well designed and executed experiments.

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Table 1. Selected Soil Properties

Property	Eudora Silt Loam	Kimo Silty Clay Loam
Textural Composition:		
sand (2.0-0.05 mm)	29.1%	20.0%
silt (0.05-0.002 mm)	54.5%	44.0%
clay (<0.002 mm)	16.4%	36.0%
Water Content at:		
-33 J/kg	0.165 g/g	0.249 g/g
-1 Kj/kg	0.061 g/g	0.140 g/g
Standard Proctor Test:		
Maximum Density	1.58 Mg/m^3	1.53 Mg/m^3
Optimum Water Content	0.155 g/g	0.192 g/g
Organic Matter:	1.50%	2.20%
Ph:	6.30	6.50
Exchangeable Cations:		
K	149 ppm	350 ppm
Ca	1698 ppm	3470 ppm
Mg	208 ppm	330 ppm
Na	8 ppm	14 ppm
Al	0 ppm	0 ppm

Table 2. A Sample Crushing Data Form

Soil type:	Silt loam								
Tillage tool:	Offset Disk								
Experiment index:	91-8-ASD111.1								
Sieve-size index (i)	0	1	2	3	4	5	6	7	8
Sieve-size x_i (mm)	0.01	0.42	0.84	2.0	6.36	19.05	44.45	76.2	152.4
ASD on mass basis:									
Before tillage ($w_0[i]$)		7.8%	2.9%	6.5%	14.9%	24.4%	25.8%	10.0%	7.6%
After tillage ($w_1[i]$)		7.9%	4.5%	10.3%	20.4%	26.3%	20.8%	6.9%	2.9%

Table 3. Parameters of Crushing Model for Four Types of Tillage Tools

Tillage Implement		Silt Loam α	Silt Loam ß	Silt Loam # data sets	Silty Clay Loam α	Silty Clay Loam ß	Silty Clay Loam # data sets
Tiller	mean	1.4	-1.2	5	1.5	0.56	3
	std. dev.	0.6	1.7		0.3	0.55	
Disk	mean	2.8	0.75	9	4.3	2.0	8
	std. dev.	0.5	0.28		1.6	1.5	
Chisel	mean	-	-	-	2.4	-2.0	4
	std. dev.	-	-		1.2	4.6	
Cultivator	mean	3.0	-0.22	1	3.0	1.8	2
	std. dev.	-	-		0.9	0.5	

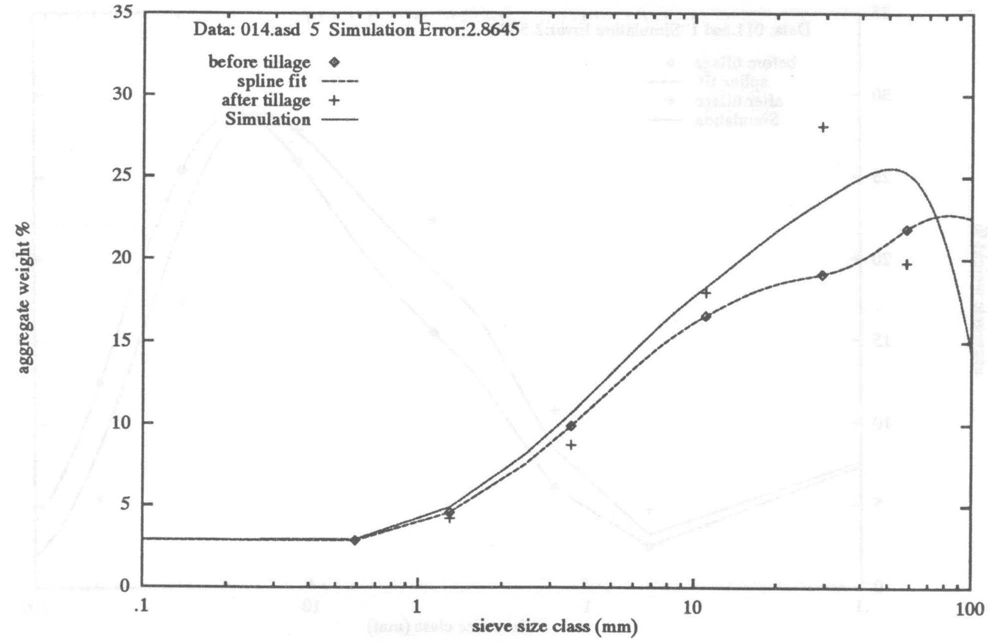

Figure 1 Crushing by the offset disk on silty clay loam with many large clods

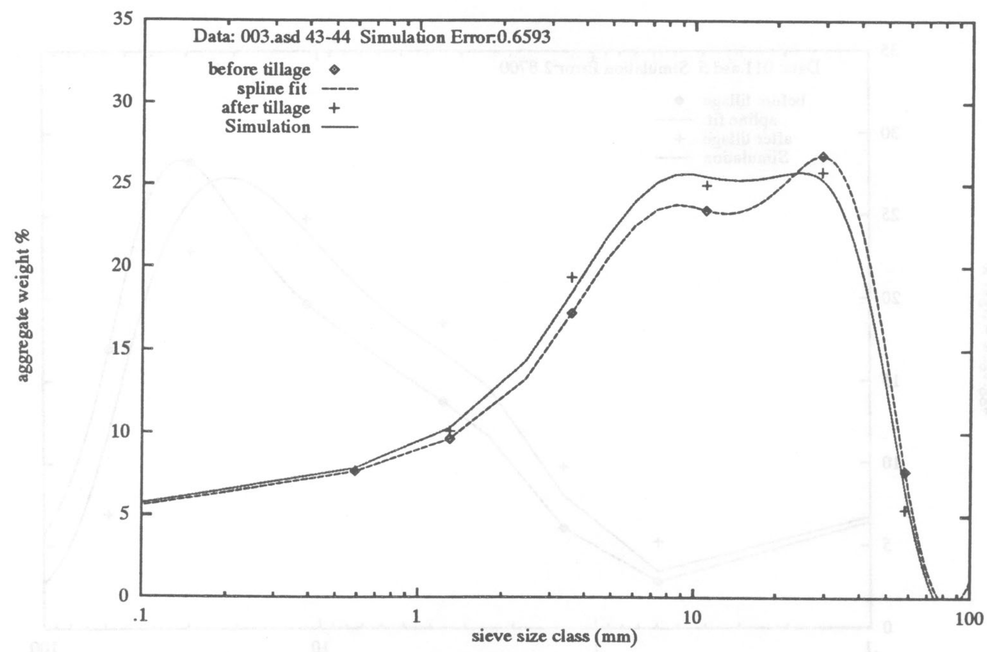

Figure 2 Crushing by the offset disk on silty clay loam with few large clods

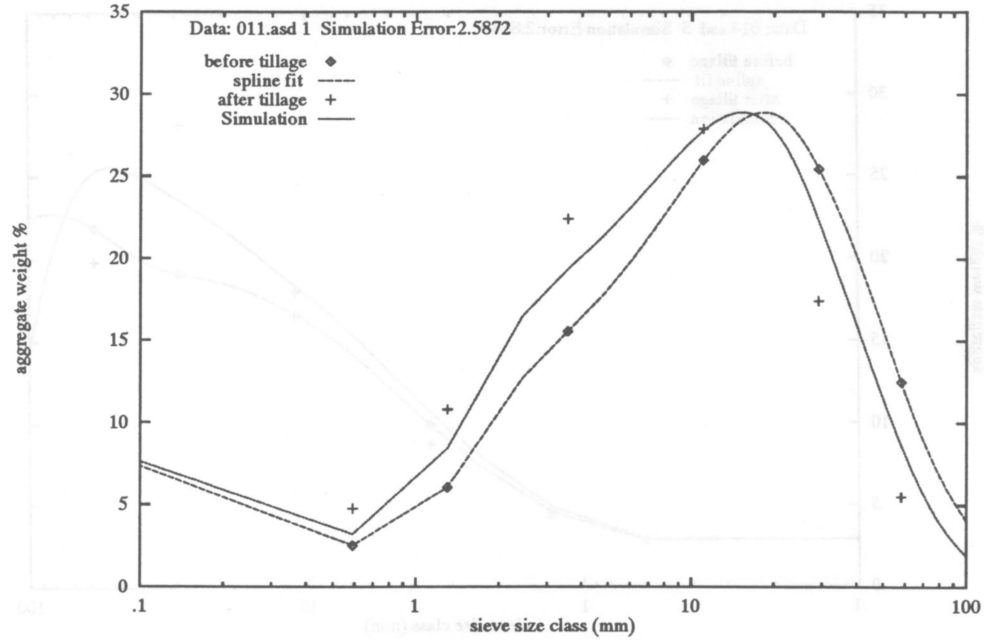

Figure 3 Crushing by the offset disk on silt loam of high moisture content

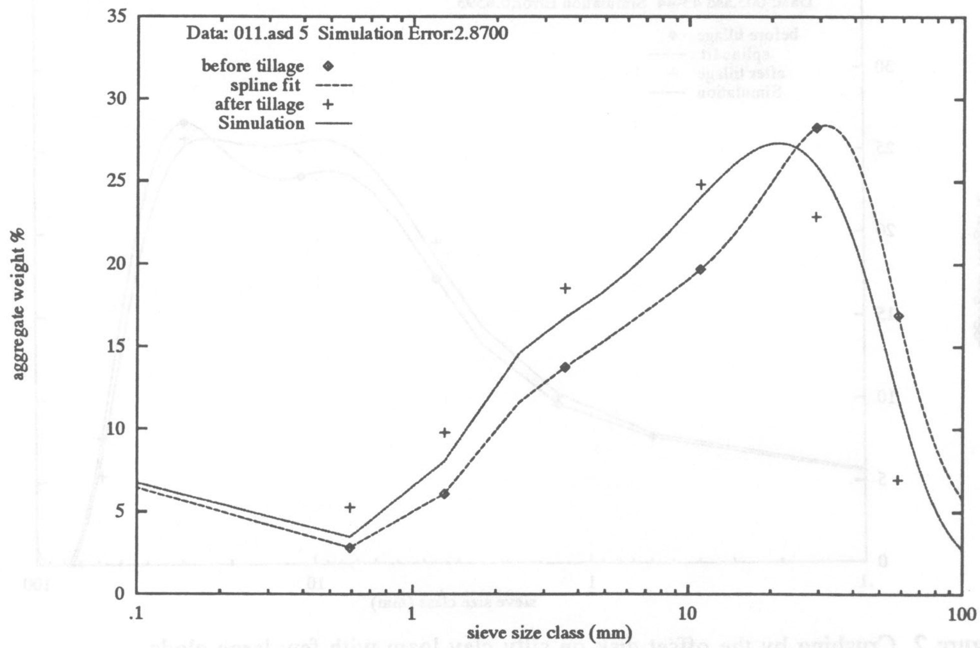

Figure 4 Crushing by the offset disk on silt loam of low moisture content