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Discussions of conservation invariably include soils and soil properties among the renewable natural resources (Chapman, 1948; Shaw, 1962). Renewal and erosion relationships, based on our national philosophy of conservation, have been presented in a paper defining soil erosion tolerance and related concepts, and providing a mathematical framework for soil conservation science (Stamey and Smith, 1964).

It has been suggested that information is adequate for use of a logical inference method guided by mathematical structure to provide useful approximate answers to erosion tolerance questions (Smith and Twiss, 1965b).

By elaboration within mathematical guidelines, the present paper aims to reconcile and clarify interrelations among soil property functions, soil management, and soil genesis; to aid recognition of pertinent data from diverse sources; and to provide hypothetical examples of certain kinds of solutions that may be helpful in applying research and experience to soil conservation problems. The particular examples are chosen to illustrate some general situations of lasting interest encompassed by experience of the authors and appropriate for detailed study in future research.

SOIL PROPERTY RENEWAL

A soil renewal function is, by definition, a nonnegative function $R(p, t)$ of position and time such that if $\alpha(p, t)$ represents the measure of any precisely defined soil property, then $R_{\alpha(p,t)} = R(\alpha(p, t), t) = R_\alpha(p, t)$ is the rate of renewal of this soil property. Thus, assuming the function R is integrable, the magnitude of renewal of this property at point p over a time interval from t_0 to $T (T > t_0)$ is given by the definite integral

$$\int_{t_0}^{T} R_\alpha(p, t)dt \qquad \text{(I)}$$

For an extensive soil property, such as depth of favorable topsoil, whose magnitude of renewal at a point is given by (I), the magnitude of renewal

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over a region A is given by $\int_A \int_{t_0}^{T} R_\alpha(p, t)dt\, dA$. Clearly, an intensive soil property such as *pH* is not integrable over a region to give magnitude.

Position and time, the first-order variables of a renewal function, are in turn functions of second-order variables of climate, crop, slope, aspect, underlying rock character, soil character, surface deposition, and perhaps others. These, in turn, are functions of a set of higher order variables. Thus, a renewal function is defined in terms of variables that are, for any order of variables, measurable and which may be integrable over time and position.

Favorable topsoil, measured in depth, may be defined as a precise soil property. As such, it is distinct from all other earth material and is subject to erosion and renewal. Assurance that topsoil is "favorable" requires restrictions on its properties. Minimal restrictions might be intermediate texture (coarse loamy, coarse silty, and fine loamy as used by the Soil Conservation Service, Soil Survey Staff, 1960) and absence of plant-toxic chemicals. Additional restrictions could include chemical, physical, and mineralogical characteristics that fit the particular situation.

One well-known means of influencing depth renewal rate through the surface deposition variable is by turning plow furrows uphill (Smith and Whitt, 1947).

Another process contributing to depth renewal rate through the surface deposition variable is sedimentation from running water or from the atmosphere. Sedimentation from running water often occurs when gradients decrease naturally or by anthropic action. Atmospheric deposition appears related, generally, to distance eastward from the "dust bowl" region of the United States (Smith and Twiss, 1965a).

A third kind of soil depth renewal is weathering of underlying rock or earth. As indicated by rock weathering studies, with climate and other variables constant, it should be possible to develop integrable functions of certain measurable characteristics of underlying rocks (Jenny, 1941; Barton, 1938; Akimtzev, 1932; Perrin, 1965) to give rates and magnitudes of soil depth renewal.

Soil Property Erosion

The consideration of a soil property erosion function $E_\alpha(p, t)$ is similar to that of a renewal function. During recent years, soil erosion has been studied in more detail than renewal.

Measurable higher order variables of time and space now being used to calculate rainfall erosion are rainfall, soil erodibility, length and steepness of slope, crop and its management, and conservation practices (Smith and Wischmeier, 1962); and to calculate wind erosion are soil erodibility, ridge roughness, climate, field length, and vegetative cover (Woodruff and Siddoway, 1965). Generally, such measured influences are integrable over time and space to give magnitudes of soil removal.

Net Change Functions

In many situations interest lies not in the gross magnitude of erosion or renewal but rather in the net change resulting from both erosion and renewal occurring simultaneously. The function of interest in this case is the net change function

$$F_\alpha(p, t) = E_\alpha(p, t) - R_\alpha(p, t) \qquad \text{(II)}$$

Magnitudes of net change are obtained by integration as illustrated for soil renewal.

Renewal, Erosion, and Net Change Functions Compared to Soil Genesis

As commonly defined, classical soil formation is a function of climate, organisms, relief, parent material, and time (Jenny, 1941; Mückenhausen, 1962; Muckenhirn, *et al.*, 1949; Soil Survey Staff, 1960). Since parent material existed only in the past, it cannot be determined precisely. Therefore, soil genesis conclusions, though useful, cannot avoid being hypothetical.

Modern soil classification recognizes that in some cases human influences determined soil properties now being used to distinguish soil individuals and mapping units (Edelman, 1950; Mückenhausen, 1962; Soil Survey Staff, 1960). Examples of beneficial and detrimental effects of man on soil-forming factors have been indicated (Bidwell and Hole, 1965). Similarly, it is essential to emphasize that variables determining soil property renewal, erosion, and net change functions are subject to human modifications. In this context it is undesirable to separate human influences as such because at some point and time every significant variable of a soil property function is expected to reflect human action.

In suggesting a complex hierarchy of influences (orders of variables), it is recognized that possible human modifications of each variable may increase or decrease this complexity. When land is managed intensively, human control now or in the future may provide an opportunity for precise determination of erosion or renewal rates that previously were not readily measurable. Accelerated renewal, like accelerated erosion, in some cases introduces rates so much greater than geologic rates that the latter may be neglected without causing serious errors.

Consideration of conservation problems may require attention only to one or a few essential soil properties rather than to the many properties determining a soil individual in a natural system of classification.

Some Choices of Functions

Erosion and Renewal Summation Over Time Only

It must be emphasized that there is a wide latitude in the choice of functions E and R applicable to measurable soil properties. One possible situation is illustrated in Figure 1. This describes graphically the behavior which might be expected when t_0 represents the time of initial cultivation of a deep soil lying over easily weathered earth or rock. The property under consideration is the depth of favorable soil material and the representation of R as an increasing function displays the increased rock-weathering rate in response to reduced depth of soil material as erosion progresses, and other increased renewal.

This example typifies situations in which satisfactory erosion control is reestablished after an interval of exhaustive land use and management. Both decreased erosion rate and increased renewal rate contribute. For a soil of assumed uniform characteristics over some region, the two functions $E(x', y', t)$ and $R(x', y', t)$ might be independent of the point (x', y') and depend only on time t.

There is, of course, an endless variation in the possible functions to describe such a situation.

One particular family of functions that might fit the erosion rate data is given by:

$$E(t) = a(t-h)e^{-b(t-h)}+c \quad \text{(III)}$$

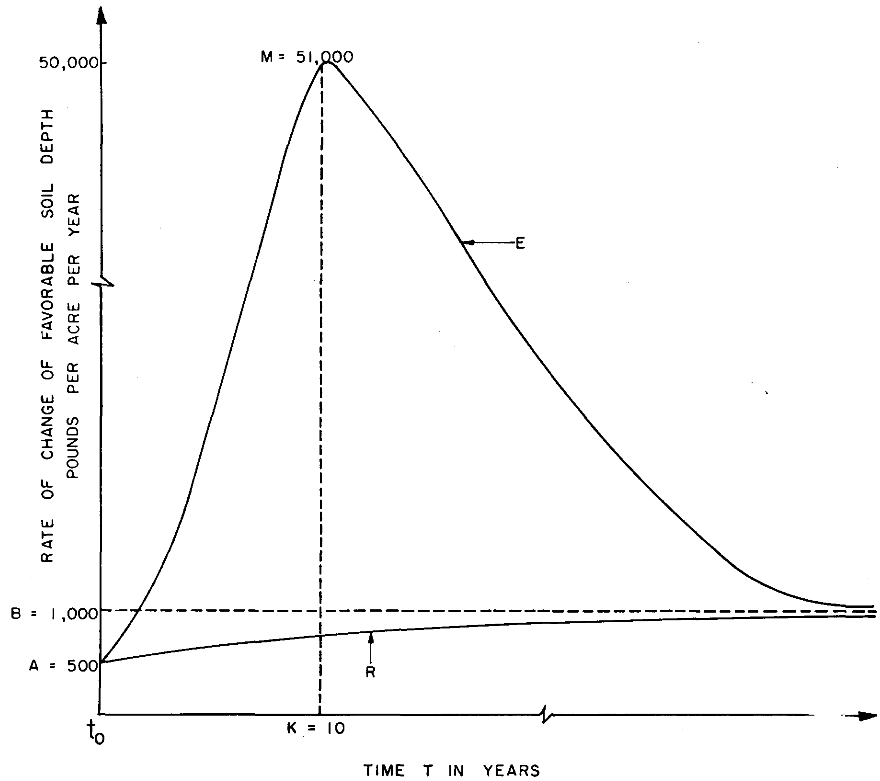

Fig. 1.—Sketch of soil erosion and renewal, typical of many situations on sloping land in central United States following initial cultivation of deep soil over easily weathered rocks.

By suitable choices of a, b, c, and h, this function can be made to assume a value A at time $t = 0$, to increase to a maximum value of $M(> A)$ at some time $t = k$, and then to decrease continually to the value $B(< M)$ as t becomes infinite.

Similarly, the renewal function might be chosen from the family of functions given by:

$$R(t) = g + me^{-nt} \tag{IV}$$

These functions assume a value of $g + m$ at time $t = 0$. For $m > 0$ they decrease continually to the value g as t becomes infinite; for $m < 0$ they increase continually to the value g. Obviously $m = 0$ gives a constant renewal rate.

Using functions from families (III) and (IV) to fit a situation similar to that illustrated in Figure 1, let $a = 5000e$, $b = \frac{1}{10}$, $c = 1000$, $h = \frac{10}{273}$, $g = 1000$, $m = -500$, and $n = 1$. Then $E(t) = 5000e(t - \frac{10}{273})e^{-\frac{1}{10}(t-\frac{10}{273})}$ $+1000 = 5000(t-\frac{10}{273})e^{1-\frac{1}{10}(t-\frac{10}{273})} + 1000 = 1000\left[1 + 5(t - \frac{10}{273})e^{\frac{2740-273t}{2730}}\right]$ and $R(t) = 1000 - 500e^{-t} = 500(2 - e^{-t})$.

Admittedly these functions and the specific values for the parameters have been chosen to give a hypothetical example which is reasonable as a physical situation and at the same time avoids excessively complicated integration and arithmetic. In future research involving greater complexity of real data mechanical details will be handled by computers.

With t measured in years, both functions represent a rate of change in soil material depth (of a suitable texture) measured in pounds per acre per year. Then the erosion and renewal both have a value of approximately 500 pounds per acre per year at time $t = 0$ years and both approach a balance of 1000 pounds per acre per year as time increases without bound. As shown in Figure 1, the renewal rate increases continually and approaches the 1000 pounds per acre per year asymptotically from below. The erosion rate increases rapidly with exhaustive cropping to a maximum of approximately 51,000 pounds per acre per year in slightly over 10 years and then decreases continually as conservation measures are improved, approaching 1000 pounds per acre per year asymptotically from above.

For these functions

$$\int_0^k (E-R)dt = \int_0^k \left[5000(t-\tfrac{10}{273})e^{\frac{2740-273t}{2730}} + 500e^{-t}\right]dt$$

$$= 500\left\{100\left[\tfrac{2720}{273}e^{\frac{274}{273}} - (k + \tfrac{2720}{273})e^{\frac{2740-273k}{2730}}\right] - e^{-k} + 1\right\}$$

Thus with $k = \infty$, $\int_0^k (E-R)dt$ is approximately 1,358,700 pounds per acre or 679 tons per acre net loss over an infinite length of time. The similar integral for finite lengths of time gives (with $k = 80$) 1,354,600 pounds per acre for 80 years and (with $k = 100$) 1,358,000 pounds per acre for 100 years. The comparison of 100 years with infinite time points up the fact that for many functions an evaluation over some reasonably long (or sometimes even relatively short) period of time is a good approximation to the improper integral.

With the assumption that $E(x', y', t)$ and $R(x', y', t)$ are independent of the point (x', y') on a particular soil mapping unit[1] in a uniformly managed field, the total loss of soil material over a region can be computed as the product of the area (in acres) times the loss per acre. This hypothetical example portrays what is meant by achieving a conservation balance at any point. If the depth of favourable soil material was sufficient at time $t = 0$, the net loss over any time interval, corresponding to the area between the curves, might be tolerable.

Net Change Summation Over Time and Space

As an example of a situation in which the functions $E(x, y, t)$ and $R(x, y, t)$ depend on the point (x, y) as well as time t, consider the following. Let the region A be a rectangular field 1,000 feet by 3,000 feet with stabilized borders along two adjacent sides as indicated in Figure 2. The area of region A is approximately 68·87 acres. Let x and y be measured in 100 feet and again let the soil property under consideration be the depth of favourable soil material. Assume that erosion control and renewal practices are to be increased over time so that the rate of erosion (principally by water action) and renewal of soil material is such that

$$F(x, y, t) = E(x, y, t) - R(x, y, t) = \frac{100x^2y}{1+t^2}$$

pounds per 10,000 square feet per year. Then the total net loss of soil material over the entire region A over an infinite time is:

$$\int_A \int_0^\infty F(x, y, t)dt\, dA = \int_0^{10} \int_0^{30} \int_0^\infty \frac{100x^2y}{1+t^2}\, dt\, dy\, dx$$

$$= \int_0^{10} \int_0^{30} 100x^2y \arctan t \Big]_0^\infty dy\, dx = \int_0^{10} \int_0^{30} 50\pi\, x^2y\, dy\, dx$$

$$= \int_0^{10} 25\pi\, x^2y^2 \Big]_0^{30} dx = \int_0^{10} 22500\pi\, x^2 dx = 7500\pi\, x^3 \Big]_0^{10} = 7500000\pi$$

which is approximately 23,550,000 pounds or approximately 171 tons per acre.

To compare the loss from each quarter of the region as indicated in Figure 2, the double integral can be evaluated separately over each subregion. In A_1 the total net loss is

[1] A mapping unit comprising a soil complex would not necessarily be an appropriate basis for a precise summation.

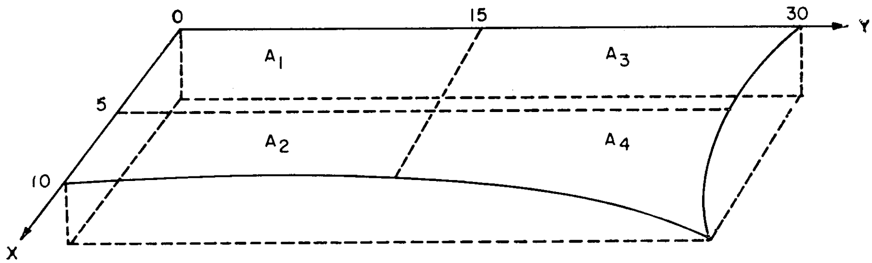

Fig. 2.—Region A: Rectangular field with stabilized borders along two adjacent sides, all sloping toward one corner; also showing subdivision of A into quadrants.

$$\int_0^5 \int_0^{15} \int_0^{\infty} \frac{100x^2y}{1+t^2}\, dt\, dy\, dx = \int_0^5 \int_0^{15} 100\, x^2y \arctan t \Big]_0^{\infty} dy\, dx$$

$$= \int_0^5 \int_0^{15} 50\pi\, x^2y\, dy\, dx = \int_0^5 25\pi\, x^2y^2 \Big]_0^{15} dx = \int_0^5 5625\pi\, x^2 dx = 1875\pi\, x^3 \Big]_0^5$$

$$= 234375\pi$$

= 735,000 pounds approximately.

In the other subregions similar integration gives

$$\int_5^{10} \int_0^{15} \int_0^{\infty} F(x, y, t)\, dt\, dy\, dx = 5{,}150{,}000 \text{ pounds approximately in } A_2;$$

in A_3 $\int_0^5 \int_{15}^{30} \int_0^{\infty} F(x, y, t)\, dt\, dy\, dx = 2{,}210{,}000$ pounds approximately; and

in A_4 $\int_5^{10} \int_{15}^{30} \int_0^{\infty} F(x, y, t)\, dt\, dy\, dx = 15{,}455{,}000$ pounds approximately.

In this case, similar computations could be made over any finite length of time as well. Increasing net loss with distances x and y could be caused by increasing $E(x, y, t)$ with $R(x, y, t)$ independent of x and y. This simple relation of erosion to x^2y could be determined by distance and steepness of slope.

Erosion of Percent Clay Over Time

Now, consider an example in which the soil property under consideration is something other than depth of favourable soil material and is intensive rather than extensive. Let the property be the percent clay (particles less than 2 microns equivalent diameter) in the top 6 inches of a level region A. Assume that leaching, weathering, etc. are balanced in their influence on clay percentage so that the only change to be considered is that due to dust deposition from the atmosphere. Suppose also that at time $t = 0$, region A is uniformly 50 percent clay and that the dust influx is regularly plowed or otherwise mixed with the top 6-inch depth of the original soil so that the uniformity of the percent clay is maintained. Then the functions

$E(x, y, t) = E(t)$ and $R(x, y, t) = R(t)$ as rates of change of percent clay are functions of time alone. Suppose further that the dust being deposited is 20 per cent clay so that the deposition represents an erosion of percent clay and that the net acre gain in soil material is approximately 500 pounds per year (the present approximate dust deposition rate at Manhattan, Kansas) (Smith and Twiss, 1965a) such that

$$F(x, y, t) = F(t) = E(t) - R(t) = \frac{12{,}000}{\pi(t^2 + 40{,}000)}$$

with t measured in decades (10 years). Then $F(t)$ is slightly less than 0·1 percent clay per decade at time $t = 0$ and decreases continually as time increases.

$$\int_0^\infty (E-R)dt = \int_0^\infty \frac{12{,}000\,dt}{\pi(t^2+40{,}000)} = \frac{60}{\pi}\arctan\frac{t}{200}\bigg]_0^\infty = \frac{60}{\pi}\cdot\frac{\pi}{2} = 30\%\text{ clay.}$$

This means that over an infinite period of time the percent clay approaches 50%−30% = 20% clay. To consider the change in percent clay over a finite period of time,

$$\int_0^{100} \frac{12{,}000\,dt}{\pi(t^2+40{,}000)} = \frac{60}{\pi}\arctan\frac{t}{200}\bigg]_0^{100} = \frac{60}{\pi}\arctan\frac{1}{2} = \frac{60}{\pi}(0\cdot4636),$$

which is approximately 8·9 percent clay, so that the percent clay after 1,000 years is 50%−8·9% = 41·1%.

This integral could also be used to answer the question "How many years are required to lower the percent clay to 33 percent?" for instance. Then the loss of percent clay is 17 percent and this gives

$$\int_0^c \frac{12{,}000\,dt}{\pi(t^2+40{,}000)} = 17$$

with c as the unknown number of years. After integrating this equation becomes $\frac{60}{\pi}\arctan\frac{c}{200} = 17$ or $c = 200\tan\frac{17\pi}{60} = 200\,(1\cdot235) = 247$ decades or 2,470 years approximately.

This particular example suggests magnitudes of textural change possibly caused by loess deposition throughout past centuries or occurring now and expected in future years.

Choosing Additional Soil Properties and Functions

The number of possible soil properties and interrelated functions is so large that considerable selectivity is necessary to assure interesting and useful choices. Even after a soil property has been decided upon and a set of data for the functions to describe it has been established, the selection of a particular function from the wide range of possibilities is a mathematical problem of some consequence.

In general, choices of properties and functions might vary widely with the individual viewpoint. However, one is not committed by conservation philosophy to include a large number of soil properties or the total soil individual. Moreover, recognition that many soil attributes are slow to change permits assumptions that they will remain unchanged over significant time intervals during which net change of a particular property is being determined in *Soil Conservation Science*.

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SUMMARY

Within mathematical structure based on American philosophy of soil conservation, soil property genesis is extended to include renewal, erosion, and net change functions of time and space as first-order variables applicable selectively to appropriately defined soil properties. Measurable variables, partially controlled by man, determine rates of erosion and renewal, often integrable to give magnitudes of change testable in terms of maintenance or improvement for intended use.

With favourable topsoil distinguished by definition from other earth material, a hypothetical example portrays typical depth change by two manipulable functions of time: erosion rate $= E(t) = a(t-h)e^{-b(t-h)}+c$ and renewal rate $= R(t) = g+me^{-nt}$.

A case of continually increasing conservation effort shows net change of favourable depth over both time t and space $x\,y$ by the function

$$F(x, y, t) = E(x, y, t)-R(x, y, t) = \frac{100x^2y}{1+t^2}.$$

A third example involves erosion (dilution) of percent clay in topsoil by atmospheric deposition where

$$F(t) = \frac{12{,}000}{\pi(t^2+40{,}000)}.$$

Soil conservation science accommodates such solutions together with theoretical soil genesis.

Résumé

Dans la structure mathématique basée sur la philosophie de conservation du sol américaine, la génération (formation) des propriétés du sol comprend le renouvellement (augmentation), l'érosion (diminution), et les fonctions de changements précis de temps et d'espace, applicables sélectivement aux propriétés du sol définies de façon appropriée. Les variables mesurables, partiellement contrôlées par l'homme, déterminent la rapidité de l'érosion et du renouvellement, souvent intégrable pour donner la grandeur de changement déterminable en terme de maintien ou amélioration pour l'utilisation prévue.

Avec un sol de surface favorable distinct par définition d'autres horizons du sol, un exemple hypothétique indique les changements typiques en profondeur par deux fonctions manipulables de temps: vitesse d'érosion $= E(t) = a(t-h)e^{-b(t-h)}+c$ et vitesse de renouvellement $= R(t) = g+me^{-nt}$.

Un cas d'effort de conservation augmentant continuellement démontre le changement précis de profondeur favorable en temps t et en espace $x\,y$ par la fonction

$$F(x, y, t) = E(x, y, t)-R(x, y, t) = \frac{100x^2y}{1+t^2}.$$

Un troisième exemple décrit l'érosion (dilution) du pourcentage d'argile du sol de surface par déposition atmosphérique où

$$F(t) = \frac{12{,}000}{\pi(t^2+40{,}000)}.$$

La science de la conservation du sol rapproche ces genres de solutions de la génération théorique du sol.

ZUSAMMENFASSUNG

Innerhalb einer mathematischen Struktur, gegründet auf die amerikanische Philosophie der Bodenkonservierung, Bodeneigenschaftsentstehung wird dahin erweitert, dass sie die Erneuerung, Erosion und absolute Wechselbeziehungen von Zeit und Raum als Variablen erster Ordnung, selektiv anwendbar zu angemessenen definierten Bodeneigenschaften, einschliesst. Messbare, teilweise menschlich regulierte Variablen, bestimmen die Geschwindigkeit der Erosion und Erneuerung, die häufig integrierbar sind, um Grössenordnungen eines Wechsels zu ergeben, die in Bezug auf Erhalt oder Verbesserung für den beabsichtigten Gebrauch geprüft werden können.

Bei günstigen Oberboden, die sich bei Definition von anderem Bodenmaterial unterscheidet, beschreibt ein hypothetisches Beispiel den typischen Tiefenwechsel durch zwei manipulierbare Funktionen der Zeit: Erosionsrate $= E(t) = a(t-h)e^{-b(t-h)}+c$ und Erneuerungsrate $= R(t) = g+me^{-nt}$.

Ein Beispiel der fortwährend steigenden Konservierungsanstrengung zeigt absolute Veränderung der günstigen Tiefe mit beidem, Zeit t und Raum $x\ y$, durch die Funktion

$$F(x, y, t) = E(x, y, t) - R(x, y, t) = \frac{100x^2y}{1+t^2}.$$

Ein drittes Beispiel zeigt die Erosion (Verdünnung) des Prozentsatzes des im oberen Boden enthaltenen Tones durch atmosphärische Ablagerung, wobei

$$F(t) = \frac{12{,}000}{\pi(t^2+40{,}000)}.$$

Die Wissenschaft der Bodenkonservierung umfasst solche mathematischen Gleichungen zusammen mit der Theorie der Entstehung des Bodens.